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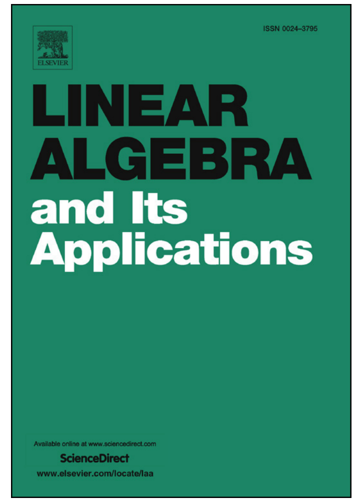

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THE POSITIVE CONTRACTIVE PART OF A NONCOMMUTATIVE L^p-SPACE IS A COMPLETE JORDAN INVARIANT

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ABSTRACT. Let $1 \leq p \leq +\infty$. We show that the positive part of the closed unit ball of a non-commutative L^p-space, as a metric space, is a complete Jordan *-invariant for the underlying von Neumann algebra.

1. INTRODUCTION

Given a von Neumann algebra M, celebrated results of R. V. Kadison showed that several partial structures of M can recover the von Neumann algebra up to Jordan *-isomorphisms. In particular, each of the following is a complete Jordan *-invariant of M: the Banach space structure of the self-adjoint part $M_{\rm sa}$ of M ([5, Theorem 2]), the ordered vector space structure of $M_{\rm sa}$ ([5, Corollary 5]) and the topological convex set structure of the normal state space of M ([6, Theorem 4.5]).

Let $p \in [1, +\infty]$, and let $L^p(M)$ be the non-commutative L^p-space associated to M with the canonical cone $L^p(M)_+$. If M is semi-finite, P.-K. Tam showed in [15] that the ordered Banach space $(L^p(M)_{\rm sa}, L^p(M)_+)$ characterises M up to Jordan *-isomorphisms. In the case when M is σ-finite (but not necessarily semi-finite) and $p = 2$, the corresponding result follows from a result of A. Connes (namely, [3, Théorème 3.3]). For a general W^*-algebra M, results of L. M. Schmitt in [13] show that the ordered Banach space $(L^p(M)_{\rm sa}, L^p(M)_+)$ determines the real Lie algebra $M/Z(M)$, where $Z(M)$ is the center of M. On the other hand, extending results of B. Russo ([12]) and F. J. Yeadon ([17]), D. Sherman showed in [14] that the Banach space $L^p(M)$ is also a complete Jordan *-invariant for a general von Neumann algebra M when $p \neq 2$.

Along these lines, we will show in this article that the underlying metric space structure of the positive contractive part

$$L^p(M)^1_+ := L^p(M)_+ \cap L^p(M)^1 \qquad (1 \leq p \leq +\infty)$$

of $L^p(M)$ is also a complete Jordan *-invariant of M, where $L^p(M)^1$ is the closed unit ball. More precisely, we obtain in Theorem 3.1 that two arbitrary von Neumann algebras M and N are Jordan *-isomorphic whenever there exists a bijection Φ from $L^p(M)^1_+$ onto $L^p(N)^1_+$ which is isometric in the sense that

$$\|\Phi(x) - \Phi(y)\| = \|x - y\| \qquad (x, y \in L^p(M)^1_+).$$

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