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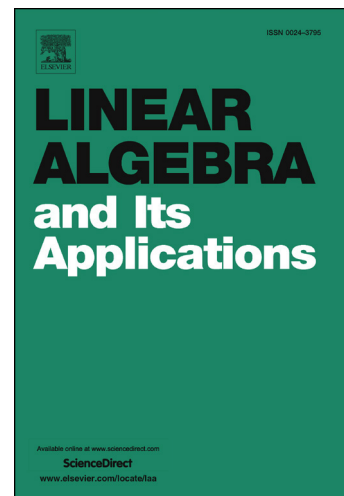
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**THE POSITIVE CONTRACTIVE PART OF A NONCOMMUTATIVE  
 $L^p$ -SPACE IS A COMPLETE JORDAN INVARIANT**

CHI-WAI LEUNG, CHI-KEUNG NG, AND NGAI-CHING WONG

ABSTRACT. Let  $1 \leq p \leq +\infty$ . We show that the positive part of the closed unit ball of a non-commutative  $L^p$ -space, as a metric space, is a complete Jordan  $*$ -invariant for the underlying von Neumann algebra.

1. INTRODUCTION

Given a von Neumann algebra  $M$ , celebrated results of R. V. Kadison showed that several partial structures of  $M$  can recover the von Neumann algebra up to Jordan  $*$ -isomorphisms. In particular, each of the following is a complete Jordan  $*$ -invariant of  $M$ : the Banach space structure of the self-adjoint part  $M_{\text{sa}}$  of  $M$  ([5, Theorem 2]), the ordered vector space structure of  $M_{\text{sa}}$  ([5, Corollary 5]) and the topological convex set structure of the normal state space of  $M$  ([6, Theorem 4.5]).

Let  $p \in [1, +\infty]$ , and let  $L^p(M)$  be the non-commutative  $L^p$ -space associated to  $M$  with the canonical cone  $L^p(M)_+$ . If  $M$  is semi-finite, P.-K. Tam showed in [15] that the ordered Banach space  $(L^p(M)_{\text{sa}}, L^p(M)_+)$  characterises  $M$  up to Jordan  $*$ -isomorphisms. In the case when  $M$  is  $\sigma$ -finite (but not necessarily semi-finite) and  $p = 2$ , the corresponding result follows from a result of A. Connes (namely, [3, Théorème 3.3]). For a general  $W^*$ -algebra  $M$ , results of L. M. Schmitt in [13] show that the ordered Banach space  $(L^p(M)_{\text{sa}}, L^p(M)_+)$  determines the real Lie algebra  $M/Z(M)$ , where  $Z(M)$  is the center of  $M$ . On the other hand, extending results of B. Russo ([12]) and F. J. Yeadon ([17]), D. Sherman showed in [14] that the Banach space  $L^p(M)$  is also a complete Jordan  $*$ -invariant for a general von Neumann algebra  $M$  when  $p \neq 2$ .

Along these lines, we will show in this article that the underlying metric space structure of the positive contractive part

$$L^p(M)_+^1 := L^p(M)_+ \cap L^p(M)^1 \quad (1 \leq p \leq +\infty)$$

of  $L^p(M)$  is also a complete Jordan  $*$ -invariant of  $M$ , where  $L^p(M)^1$  is the closed unit ball. More precisely, we obtain in Theorem 3.1 that two arbitrary von Neumann algebras  $M$  and  $N$  are Jordan  $*$ -isomorphic whenever there exists a bijection  $\Phi$  from  $L^p(M)_+^1$  onto  $L^p(N)_+^1$  which is isometric in the sense that

$$\|\Phi(x) - \Phi(y)\| = \|x - y\| \quad (x, y \in L^p(M)_+^1).$$

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