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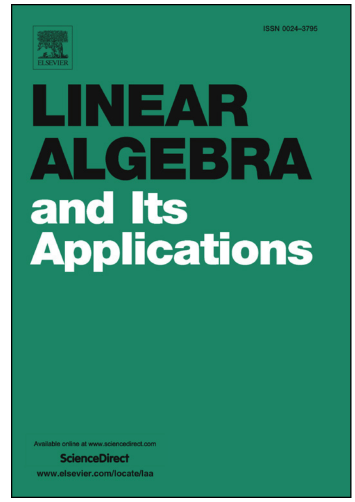

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Maximizing the least signless Laplacian eigenvalue of unicyclic graphs*

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Abstract In this paper, a graph with the maximum least signless Laplacian eigenvalue among all connected unicyclic graphs with fixed order is determined.

AMS classification: 05C50.

Keywords: Signless Laplacian matrix; signless Laplacian characteristic polynomial; the least signless Laplacian eigenvalue; eigenvector

1 Introduction

Let $G = (V, E)$ be a simple connected graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set E. Its adjacency matrix $A(G) = (a_{ij})$ is defined as $n \times n$ matrix (a_{ij}), where $a_{ij} = 1$ if v_i is adjacent to v_j; and $a_{ij} = 0$, otherwise. Denote by $d(v_i)$ or $d_G(v_i)$ the degree of the vertex v_i. The matrices $L(G) = D(G)-A(G)$ and $Q(G) = D(G) + A(G)$ are called the Laplacian matrix and the signless Laplacian matrix of graph G, respectively, where $D(G) = diag(d(v_1), d(v_2), \cdots, d(v_n))$ denotes the diagonal matrix of vertex degrees of G. It is easy to see that $L(G)$ is a positive semidefinite symmetric matrix with the smallest eigenvalue 0 and the corresponding eigenvector is the column vector of all ones. Fiedler [10] showed that the second smallest eigenvalue of $L(G)$ is 0 if and only if G is disconnected. Thus the second smallest eigenvalue of $L(G)$ is popularly known as the *algebraic connectivity* of G.

It is well known that $A(G)$ is a real symmetric matrix and $Q(G)$ is a positive semidefinite symmetric matrix. The eigenvalues of $Q(G)$ can be ordered as

$$q_1(G) \geq q_2(G) \geq \cdots \geq q_n(G) \geq 0,$$

respectively. The smallest eigenvalue $q_n(G)$ of $Q(G)$ is called the least signless Laplacian eigenvalue of the graph G, denoted by $q(G)$. It is a well known fact that for a connected graph G, $q(G) = 0$ if and only if G is bipartite [3]. A pendant vertex is a vertex with degree one. If v is a pendant vertex and v is adjacent to u, then v is called the pendant vertex of u.

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