

Accepted Manuscript

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PII: S0024-3795(17)30023-X
DOI: <http://dx.doi.org/10.1016/j.laa.2017.01.007>
Reference: LAA 14008

To appear in: *Linear Algebra and its Applications*

Received date: 13 April 2016
Accepted date: 10 January 2017

Please cite this article in press as: P. Benito et al., Free nilpotent and nilpotent quadratic Lie algebras, *Linear Algebra Appl.* (2017), <http://dx.doi.org/10.1016/j.laa.2017.01.007>

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Abstract

In this paper we introduce an equivalence between the category of the t -nilpotent quadratic Lie algebras with d generators and the category of some symmetric invariant bilinear forms over the t -nilpotent free Lie algebra with d generators. Taking into account this equivalence, t -nilpotent quadratic Lie algebras with d generators are classified (up to isometric isomorphisms, and over any field of characteristic zero), in the following cases: $d = 2$ and $t \leq 5$, $d = 3$ and $t \leq 3$.

Keywords: Nilpotent Lie algebras, invariant nondegenerate symmetric bilinear forms, free nilpotent Lie algebras.

Classification MSC 2010: 17B01, 17B30

1 Introduction

Let \mathfrak{n} be a Lie algebra over an arbitrary field \mathbb{K} of characteristic zero. The Lie algebra \mathfrak{n} is said to be nilpotent if $\mathfrak{n}^{t+1} = 0$, where \mathfrak{n}^t is defined inductively as $\mathfrak{n}^1 = \mathfrak{n}$, $\mathfrak{n}^i = [\mathfrak{n}^{i-1}, \mathfrak{n}]$. In this case we call t the index of nilpotency of \mathfrak{n} and we say that \mathfrak{n} is t -nilpotent or also t -step nilpotent ($\mathfrak{n}^t \neq 0$). The chain of ideals of \mathfrak{n} :

$$\mathfrak{n} \supseteq \mathfrak{n}^2 \supseteq \dots \supseteq \mathfrak{n}^t \supseteq \mathfrak{n}^{t+1} \supseteq \dots \quad (1)$$

is the well-known lower central series of \mathfrak{n} . Hence, if \mathfrak{n} is t -nilpotent the lower central series finishes after $t + 1$ steps.

The type of a nilpotent Lie algebra \mathfrak{n} is defined as the codimension of \mathfrak{n}^2 in \mathfrak{n} . Following M. A. Gauger [5, Section 1, Corollary 1.3], a set $\mathfrak{m} = \{x_1, x_2, \dots, x_d\}$ generates \mathfrak{n} if and only if $\{x_1 + \mathfrak{n}^2, \dots, x_d + \mathfrak{n}^2\}$ is a basis of $\mathfrak{n}/\mathfrak{n}^2$. So, the type of a Lie algebra is the cardinal of every \mathbb{K} -linearly independent set, $\mathfrak{m} = \{x_1, x_2, \dots, x_d\}$, such that \mathfrak{t} , the subspace generated by \mathfrak{m} , satisfies $\mathfrak{t} \oplus \mathfrak{n}^2 = \mathfrak{n}$. The above conditions imply that $\mathfrak{m} = \{x_1, \dots, x_d\}$ generates \mathfrak{n} as \mathbb{K} -algebra and therefore, we can see the elements $x_i \in \mathfrak{m}$ as a *minimal set of generators* of \mathfrak{n} .

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