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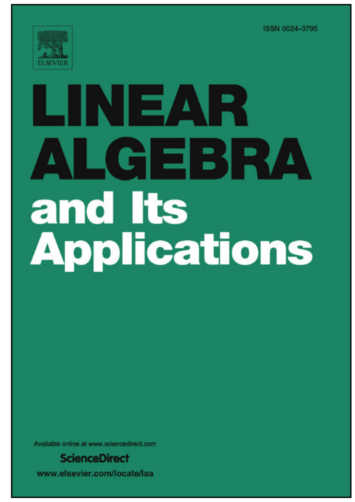

Free nilpotent and nilpotent quadratic Lie algebras

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Abstract

In this paper we introduce an equivalence between the category of the t-nilpotent quadratic Lie algebras with d generators and the category of some symmetric invariant bilinear forms over the t-nilpotent free Lie algebra with d generators. Taking into account this equivalence, t-nilpotent quadratic Lie algebras with d generators are classified (up to isometric isomorphisms, and over any field of characteristic zero), in the following cases: $d = 2$ and $t \leq 5$, $d = 3$ and $t \leq 3$.

Keywords: Nilpotent Lie algebras, invariant nondegenerate symmetric bilinear forms, free nilpotent Lie algebras.

Classification MSC 2010: 17B01, 17B30

1 Introduction

Let $\mathfrak{n}$ be a Lie algebra over an arbitrary field $\mathbb{K}$ of characteristic zero. The Lie algebra $\mathfrak{n}$ is said to be nilpotent if $\mathfrak{n}^{t+1} = 0$, where $\mathfrak{n}^t$ is defined inductively as $\mathfrak{n}^1 = \mathfrak{n}$, $\mathfrak{n}^i = [\mathfrak{n}^{i-1}, \mathfrak{n}]$. In this case we call t the index of nilpotency of $\mathfrak{n}$ and we say that $\mathfrak{n}$ is t-nilpotent or also t-step nilpotent ($\mathfrak{n}^t \neq 0$). The chain of ideals of $\mathfrak{n}$:

$$\mathfrak{n} \supseteq \mathfrak{n}^2 \supseteq \cdots \supseteq \mathfrak{n}^t \supseteq \mathfrak{n}^{t+1} \supseteq \dots \tag{1}$$

is the well-known lower central series of $\mathfrak{n}$. Hence, if $\mathfrak{n}$ is t-nilpotent the lower central series finishes after $t + 1$ steps.

The type of a nilpotent Lie algebra $\mathfrak{n}$ is defined as the codimension of $\mathfrak{n}^2$ in $\mathfrak{n}$. Following M. A. Gauger [5, Section 1, Corollary 1.3], a set $\mathfrak{m} = \{x_1, x_2, \dots, x_d\}$ generates $\mathfrak{n}$ if and only if $\{x_1 + \mathfrak{n}^2, \dots x_d + \mathfrak{n}^2\}$ is a basis of $\mathfrak{n}/\mathfrak{n}^2$. So, the type of a Lie algebra is the cardinal of every $\mathbb{K}$-linearly independent set, $\mathfrak{m} = \{x_1, x_2, \dots x_d\}$, such that $\mathfrak{t}$, the subspace generated by $\mathfrak{m}$, satisfies $\mathfrak{t} \oplus \mathfrak{n}^2 = \mathfrak{n}$. The above conditions imply that $\mathfrak{m} = \{x_1, \dots, x_d\}$ generates $\mathfrak{n}$ as $\mathbb{K}$-algebra and therefore, we can see the elements $x_i \in \mathfrak{m}$ as a *minimal set of generators* of $\mathfrak{n}$.