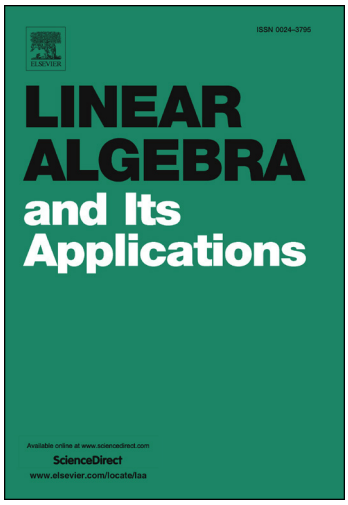

On the permanents of circulant and degenerate Schur matrices

Vitaly V. Kocharovsky, Vladimir V. Kocharovsky

PII: S0024-3795(17)30053-8
DOI: http://dx.doi.org/10.1016/j.laa.2017.01.024
Reference: LAA 14025

To appear in: *Linear Algebra and its Applications*

Received date: 9 October 2016
Accepted date: 17 January 2017

Please cite this article in press as: V.V. Kocharovsky, Vl.V. Kocharovsky, On the permanents of circulant and degenerate Schur matrices, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.01.024

On the permanents of circulant and degenerate Schur matrices

Vitaly V. Kocharovsky[a,b,*], Vladimir V. Kocharovsky[b]

[a] *Department of Physics and Astronomy, Texas A&M University, College Station, TX 77843-4242, USA*
[b] *Institute of Applied Physics, Russian Academy of Science, 603950 Nizhny Novgorod, Russia*

Abstract

We communicate three formulas for the permanents of degenerate Schur and circulant matrices. These combinatorial and integral formulas are intended for the analytical and asymptotic evaluation of the permanents as well as for the solution of three-dimensional Ising model. The paper's goal is to draw attention to the open fundamental problem of finding the permanents' asymptotics. A solution to this problem would be tremendously important for physics of many-body systems and critical phenomena, as well as for quantum field theory.

Keywords: Permanent, Degenerate Schur matrix, Circulant matrix, Multiset partition, q-function, Ising model
2010 MSC: 15A15, 15B05, 05A18, 05A30, 11D04, 11D45, 82B20

1. Introduction. Significance and complexity of permanents

The permanent, per$A$, and the determinant, det$A$, of a $n \times n$ matrix A correspond to two major operations - the symmetrization and the anti-symmetrization, respectively. This fact predetermines their fundamental role in the quantum theory of many-body systems which are either bosonic (symmetric) or fermionic (anti-symmetric). The permanents are well-known in mathematical physics, especially, in quantum computing science and quantum field theory of interacting Bose fields [5, 36, 26, 1, 28, 8]. However, compared to the determinants, the permanents are much more difficult to compute and they account for much more complicated many-body phenomena, such as the critical phenomena in phase transitions. In particular, the exact general solution of a long-standing three-dimensional (3D) Ising model has been found recently in terms of the permanents [18, 19, 20]. This solution includes a remarkable compact formula for

*Corresponding author.
E-mail addresses: vkochar@physics.tamu.edu (V.V. Kocharovsky), kochar@appl.sci-nnov.ru (Vl.V. Kocharovsky).