

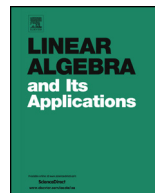


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Inequalities related to partial transpose and partial trace



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ABSTRACT

In this paper, we present inequalities related to partial transpose and partial trace for positive semidefinite matrices. Some interesting results involving traces and eigenvalues are also included.

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1. Introduction

Throughout the paper, we use the following standard notation:

- $\mathbb{M}_{n \times k}$ is the set of $n \times k$ complex matrices; if $n = k$, we use \mathbb{M}_n for $\mathbb{M}_{n \times n}$ and if $k = 1$, we use \mathbb{C}^n for $\mathbb{M}_{n \times 1}$.
- $\mathbb{M}_n(\mathbb{M}_k)$ is the set of $n \times n$ block matrices with each block in \mathbb{M}_k .

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- I_n is the $n \times n$ identity matrix.
- $A \circ B$ is the Hadamard product of A, B .
- $A \otimes B$ is the Kronecker product of A, B ; that is, if $A = [a_{ij}] \in \mathbb{M}_n$ and $B \in \mathbb{M}_k$, then $A \otimes B \in \mathbb{M}_n(\mathbb{M}_k)$ whose (i, j) block is $a_{ij}B$.
- $\|x\|$ denotes the 2-norm of $x \in \mathbb{C}^n$; that is, $\|x\| = \sqrt{x^*x}$.

Given $A = [A_{i,j}]_{i,j=1}^n \in \mathbb{M}_n(\mathbb{M}_k)$, we define the partial transpose of A by

$$A^\tau = [A_{j,i}]_{i,j=1}^n.$$

Note that $A \geq 0$ does not necessarily imply $A^\tau \geq 0$. If both A and A^τ are positive semidefinite, then A is said to be positive partial transpose (PPT for short). We also define two partial traces $\text{tr}_1 A$ and $\text{tr}_2 A$ of $A = [A_{i,j}]_{i,j=1}^n \in \mathbb{M}_n(\mathbb{M}_k)$ by

$$\begin{aligned} \text{tr}_1 A &= \sum_{i=1}^n A_{i,i}, \\ \text{tr}_2 A &= [\text{tr} A_{i,j}]_{i,j=1}^n, \end{aligned}$$

where $\text{tr} X$ denotes the trace of X ; see [5] for more details related to partial traces.

It is shown in [1,4] that for any positive semidefinite $H \in \mathbb{M}_n(\mathbb{M}_k)$, we have

$$I_n \otimes \text{tr}_1 H + \text{tr}_2 H \otimes I_k - H \leq (\text{tr} H) I_{nk}. \tag{1.1}$$

Generally, $I_n \otimes \text{tr}_1 H + \text{tr}_2 H \otimes I_k \geq H$ does not hold. In this paper, we show

$$I_n \otimes \text{tr}_1 H + \text{tr}_2(H^\tau) \otimes I_k \geq H^\tau$$

for $H \in \mathbb{M}_n(\mathbb{M}_k)$ with $H \geq 0$ (in Theorem 2) and

$$I_2 \otimes \text{tr}_1 H + \text{tr}_2(H) \otimes I_k \geq H$$

for $H \in \mathbb{M}_2(\mathbb{M}_k)$ with $H \geq 0$ (in Theorem 4). Moreover, some interesting inequalities involving the trace and the minimum and maximum eigenvalues of a positive semidefinite matrix will be presented.

2. Results and proofs

Lemma 1. *Let $A, B \in \mathbb{M}_n$. If $A, B \geq 0$, then $\sum(A \circ B) \geq 0$, where $\sum(X)$ denotes the sum of all entries of X .*

Proof. By the Schur product theorem, we have $A \circ B \geq 0$. Since $\sum(X) = u^* X u$, where $u \in \mathbb{C}^n$ is the vector with all entries equal to one, $\sum(A \circ B) \geq 0$ follows from $A \circ B \geq 0$. \square

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