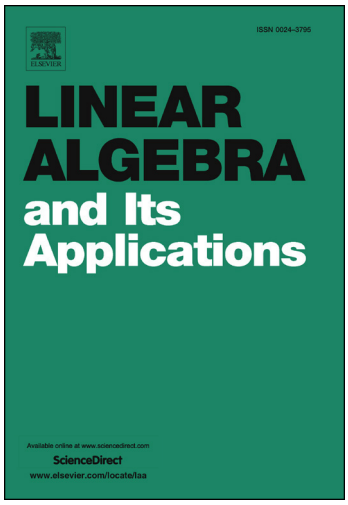

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Sets of refined inertias of zero-nonzero patterns

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Abstract

A set $\mathbb{H}_n^*$ of refined inertias for zero-nonzero patterns is introduced that is analogous to the set $\mathbb{H}_n$ previously considered for sign patterns. For $n = 3$ and 4, a complete characterization of irreducible zero-nonzero patterns that allow or require $\mathbb{H}_n^*$ is given, and each zero-nonzero pattern that allows $\mathbb{H}_n^*$ has a signing that allows $\mathbb{H}_n$. In contrast, for $n \geq 5$ a family of irreducible zero-nonzero patterns is given that allows $\mathbb{H}_n^*$ but for which no one signing allows $\mathbb{H}_n$.

Keywords: zero-nonzero pattern, sign pattern, refined inertia, eigenvalues, digraph

2010 MSC: 15B35, 15A18, 05C50, 05C20

1. Introduction

The set of refined inertias $\mathbb{H}_n$ is relevant to the study of dynamical systems, where the presence of nonzero pure imaginary eigenvalues can signal the onset of periodic solutions by Hopf bifurcation (see [1]). Previous papers (e.g., [1, 2, 3, 4, 5]) have focused on sign patterns that require or allow $\mathbb{H}_n$. Here the concepts in these papers are extended to zero-nonzero patterns. In particular, the set of refined inertias $\mathbb{H}_n$ is expanded to a set $\mathbb{H}_n^*$, and we show that, for $n \geq 5$, there exist $n \times n$ zero-nonzero patterns that allow $\mathbb{H}_n^*$ and for which there does not exist a signing of these zero-nonzero patterns that allows $\mathbb{H}_n$. The implication of this is that there may be a difference in the use of refined inertias in the