

# Accepted Manuscript

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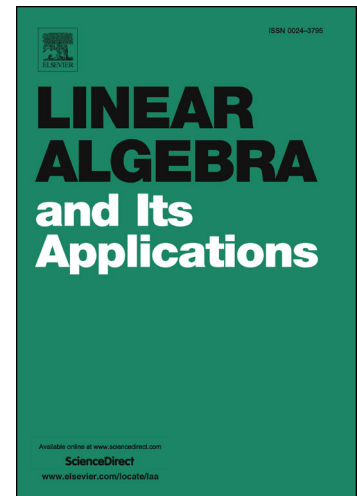
PII: S0024-3795(17)30178-7  
DOI: <http://dx.doi.org/10.1016/j.laa.2017.03.016>  
Reference: LAA 14092

To appear in: *Linear Algebra and its Applications*

Received date: 8 March 2017  
Accepted date: 16 March 2017

Please cite this article in press as: S.R. Garcia, D. Sherman, Matrices similar to partial isometries, *Linear Algebra Appl.* (2017), <http://dx.doi.org/10.1016/j.laa.2017.03.016>

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## MATRICES SIMILAR TO PARTIAL ISOMETRIES

STEPHAN RAMON GARCIA AND DAVID SHERMAN

ABSTRACT. We determine when a matrix is similar to a partial isometry, refining a result of Halmos–McLaughlin.

## 1. INTRODUCTION

A Hilbert space operator  $V$  is a *partial isometry* if the restriction of  $V$  to  $(\ker V)^\perp$  is isometric. For a complex matrix, this means that all of its singular values are in  $\{0, 1\}$ , or in other words, the positive semidefinite factor in its polar decomposition is an orthogonal projection. These properties are not preserved by similarity; for example

$$\begin{bmatrix} 0 & \frac{\sqrt{3}}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \quad (1)$$

are similar, since both matrices have the same Jordan canonical form. The first is a partial isometry since its nonzero columns are orthonormal, but the second is not. Which matrices are similar to a partial isometry?

The basic features of partial isometries were laid out over fifty years ago [2,6,7], and the similarity question is not new – but most work has focused on the (still unresolved) infinite-dimensional case, e.g., [3]. In the finite-dimensional case, the best result was a theorem of Halmos–McLaughlin stating that the characteristic polynomial of a nonunitary partial isometry can be any monic polynomial whose roots lie in the closed unit disk and include zero [6, Theorem 3]. The referee pointed out that this can be deduced directly from the Weyl–Horn inequalities [8,13], which say that there exists an  $n \times n$  matrix with prescribed singular values  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0$  and eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$ , indexed so that  $|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_n|$ , if and only if

$$\sigma_1 \sigma_2 \cdots \sigma_k \geq |\lambda_1 \lambda_2 \cdots \lambda_k| \quad \text{for } k = 1, 2, \dots, n-1$$

and

$$\sigma_1 \sigma_2 \cdots \sigma_n = |\lambda_1 \lambda_2 \cdots \lambda_n|.$$

For an  $n \times n$  partial isometry of rank  $r < n$ , we have

$$\sigma_1 = \sigma_2 = \cdots = \sigma_r = 1 \quad \text{and} \quad \sigma_{r+1} = \cdots = \sigma_n = 0.$$

Hence any  $n$  points (with possible repetition) in the closed unit disk can be its eigenvalues, so long as at least  $n - r$  of them are zero.

Date: March 17, 2017.

2010 *Mathematics Subject Classification*. 15A21.

*Key words and phrases*. Partial isometry; partially isometric matrix; similarity; Jordan form.

The authors acknowledge the support of NSF grants DMS-1265973 and DMS-1201454.

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