Linear Algebra and its Applications 529 (2017) 12–29

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Perturbation bounds for Mostow's decomposition and the bipolar decomposition

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ARTICLE INFO

Article history:
Received 21 December 2016
Accepted 30 March 2017
Available online xxxx
Submitted by P. Semrl

MSC:
15A23
47A55
65F60
47A64
47A30
15A45

Keywords:
Perturbation bounds
Derivative
Matrix factorizations
The bipolar decomposition
Mostow's decomposition theorem
Polar decomposition
Sylvester's equation
Geometric mean

ABSTRACT

Perturbation bounds for Mostow's decomposition and the bipolar decomposition of matrices have been computed. To do so, expressions for the derivative of the geometric mean of two positive definite matrices have been derived.

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http://dx.doi.org/10.1016/j.laa.2017.03.027

1. Introduction

Matrix factorizations have been used in numerical analysis to implement efficient matrix algorithms. In machine learning, matrix factorizations play an important role to explain latent features underlying the interactions between different kinds of entities. Many matrix factorizations namely, the polar decomposition, the QR decomposition, the LR decomposition etc., have been of considerable interest for many decades. Perturbation bounds for such factorizations have been of interest for a long time (see [2,21,22] and the references therein). Some generalizations and improvements on them have been obtained in the subsequent works, for example, see [11–13,16–19,23].

An interesting matrix factorization follows from the work of Mostow [20]. It states that every nonsingular complex matrix Z can be uniquely factorized as

$$Z = We^{iK}e^{S}, \tag{1.1}$$

where W is a unitary matrix, S is a real symmetric matrix and K is a real skew symmetric matrix. Recently, Bhatia [6] showed that every complex unitary matrix W can be factorized as

$$W = e^{L}e^{iT}, \tag{1.2}$$

where L is a real skew symmetric matrix and T is a real symmetric matrix. Using (1.1) and (1.2), it has been obtained in [6] that

$$Z = e^{L}e^{iT}e^{iK}e^{S}. \tag{1.3}$$

Our goal is to find the perturbation bounds for the factors arising in (1.1), (1.2) and (1.3). In [1], Barbaresco has used Berger Fibration in Unit Siegel Disk for Radar Space–Time Adaptive Processing and Toeplitz–Block–Toeplitz covariance matrices based on Mostow's decomposition.

Let $\mathbb{M}(n, \mathbb{C})$ be the space of $n \times n$ complex matrices, and let $\mathbb{U}(n, \mathbb{C})$ be the set of $n \times n$ complex unitary matrices. Let $|||\cdot|||$ be any unitarily invariant norm on $\mathbb{M}(n, \mathbb{C})$, that is, for any $U, V \in \mathbb{U}(n, \mathbb{C})$ and $A \in \mathbb{M}(n, \mathbb{C})$ we have

$$|||UAV||| = |||A|||.$$

Two special examples of such norms are the *operator norm* $\|\cdot\|$ (also known as the *spectral norm*) and *Frobenius norm* $\|\cdot\|_2$ (also known as *Hilbert–Schmidt norm* or *Schatten* 2-*norm*). Various properties of unitarily invariant norms are known [3, Chapter IV]. We would require the following important properties: for $A, B, C \in \mathbb{M}(n, \mathbb{C})$

$$|||ABC||| \leq \|A\| \; |||B||| \; \|C\|, \tag{1.4}$$

and