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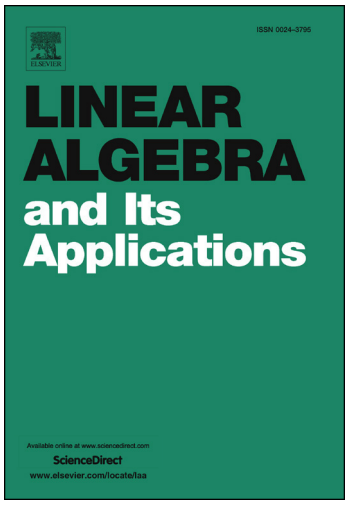

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A Connection Between Hadamard Matrices, Oriented Hypergraphs and Signed Graphs

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Abstract

Matrices associated to oriented hypergraphs produce a connection between signed graphs and Hadamard matrices. The existence of a family of signed graphs that are switching equivalent to $-K_n$ and whose adjacency matrices sum to the zero matrix is shown to be equivalent to the existence of a Hadamard matrix. This equivalent problem is used to make explicit signed graph constructions which specialize to known Hadamard constructions.

Keywords: Hadamard matrix, oriented hypergraph, signed graph
2010 MSC: Primary: 05C50, Secondary: 05C65, 05C22

1. Introduction

An *oriented hypergraph* is a hypergraph where each vertex-edge incidence is given a label of either $+1$ or -1 [8, 9]. Oriented hypergraphs, a hypergraph generalization of oriented signed graphs [14], were independently developed by Shi who called them *signed hypergraphs* [12, 11]. The vertex-edge labeling of an oriented hypergraph provides a natural definition for a *signed adjacency* that is used to define the adjacency and Laplacian matrices [2, 6, 8]. Recently the study of these associated matrices has led to a connection between oriented hypergraphs, signed graphs and balanced incomplete block designs (BIBDs) [7].

An $n \times n$ matrix $\mathfrak{H}$ whose entries are either $+1$ or -1 is called a *Hadamard matrix of order* n if $\mathfrak{H}\mathfrak{H}^{\mathrm{T}} = nI$. It is easy to show that for all Hadamard matrices, n must be either 1,2 or a positive multiple of 4. The question of existence is much harder, and the longstanding Hadamard conjecture states that a Hadamard matrix of order n always exists when n is divisible by 4.

In this paper, oriented hypergraphs are used to form a relationship between a family of signed graphs and Hadamard matrices. The existence of this family allows one to construct a Hadamard matrix, and conversely, a Hadamard matrix produces such a family. Using this equivalent problem, we can recover some known constructions for Hadamard matrices in the context of oriented hypergraphs and signed graphs.

In Section 2, relevant background on oriented hypergraphs, signed graphs, matrices and switching is provided. In Section 3, the main theorem of the paper is presented, where the existence of a Hadamard matrix is shown to be equivalent to the existence of a special collection of signed graphs. In Section 4 the main theorem is used to construct a Hadamard matrix. Known constructions of Hadamard matrices are independently developed using the equivalent family of special signed graphs.

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