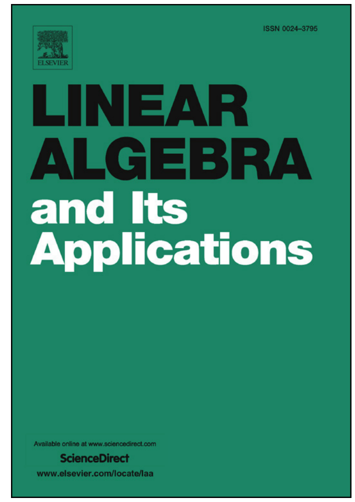

Nilpotent Bridging for Unions of Two Bases

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NILPOTENT BRIDGING FOR UNIONS OF TWO BASES

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Abstract. In a recent article, the authors discovered a new, efficient method for perfect reconstruction from frame erasures called *nilpotent bridging*. In order to perform the reconstruction, it is necessary to invert a matrix, called the *bridge matrix*, which consists of inner products of the frame vectors indexed by the *erasure set*, with vectors from the corresponding dual frame indexed by a set known as the *bridge set*, which is disjoint from the erasure set. If no search procedure is required to find a bridge set of the same size as the erasure set for which the bridge matrix is invertible, the dual frame pair is said to satisfy the *full skew-spark property*. Equivalently, a frame satisfies the full skew-spark property if every subset of the complement of the erasure set with the same cardinality as the erasure set yields an invertible bridge matrix. In this paper, we consider dual frame pairs which consist of a union of two bases, and the union of their corresponding dual bases. In finite dimensions, it is shown that if we restrict the class of bridge sets to *block bridge sets*, then a bridge set search is also unnecessary for most unions of two bases, and most unions of two orthonormal bases. By most, we mean the set of unions of two bases (resp. orthonormal bases) for which no bridge set search is necessary contains an open, dense set in the set of all unions of two bases (resp. orthonormal bases). So, while not every bridge set of the same cardinality as the erasure set yields an invertible bridge matrix, the bridge sets which have the special structure of block bridge sets are very likely to yield an invertible bridge matrix. These unions of two bases are said to satisfy the *block skew-spark property*. In infinite dimensions, we also show that the block skew-spark property holds for a dense (but not necessarily open) set of unions of two Riesz (resp. orthonormal) bases. Lastly, applications to Shannon-Whittaker sampling theory are discussed, and several open questions are posed pertaining to sampling theory.

1. Introduction

In [13] the authors of this paper discovered a new method of reconstruction from frame erasures that was more efficient than older methods in the literature. This method, called *nilpotent bridging* allows for a perfect reconstruction from frame erasures by inverting a matrix of size $L \times L$ where L is the size of the erased set of indices, Λ, whereas traditional methods of reconstruction required a matrix inversion of size $n \times n$, where n denotes the dimension of the underlying Hilbert space. To achieve a reconstruction, this method uses a subset of the non-erased frame coefficients, $\Omega \subset \Lambda^c$ known as the *bridge set*, to "support" the erased data. Given a dual frame pair $\{f_j, g_j\}_{j\in\mathbb{J}}$, if the *bridge equation*:

$$(\langle f_j, g_k\rangle)_{j\in\Lambda,\, k\in\Omega}\,(\overline{c_{j,k}})_{j\in\Omega,\, k\in\Lambda} = (\langle f_j, g_k\rangle)_{j,k\in\Lambda}$$

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