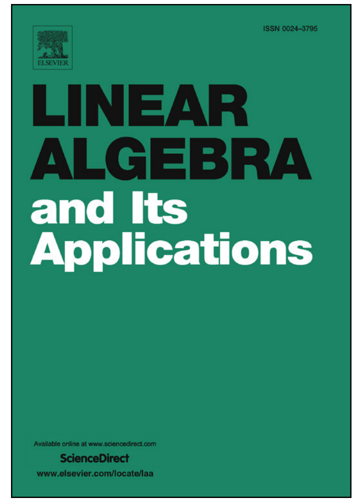
Accepted Manuscript

Combinatorial methods for the spectral p-norm of hypermatrices

V. Nikiforov



PII: S0024-3795(17)30260-4
DOI: http://dx.doi.org/10.1016/j.laa.2017.04.023
Reference: LAA 14132

To appear in: *Linear Algebra and its Applications*

Received date: 11 July 2016
Accepted date: 21 April 2017

Please cite this article in press as: V. Nikiforov, Combinatorial methods for the spectral p-norm of hypermatrices, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.04.023

Combinatorial methods for the spectral p-norm of hypermatrices

V. Nikiforov*

Abstract

The spectral p-norm of r-matrices generalizes the spectral 2-norm of 2-matrices. In 1911 Schur gave an upper bound on the spectral 2-norm of 2-matrices, which was extended in 1934 by Hardy, Littlewood, and Pólya to r-matrices. Recently, Kolotilina, and independently the author, strengthened Schur's bound for 2-matrices. The main result of this paper extends the latter result to r-matrices, thereby improving the result of Hardy, Littlewood, and Pólya.

The proof is based on combinatorial concepts like *r-partite r-matrix* and *symmetrant* of a matrix, which appear to be instrumental in the study of the spectral p-norm in general. Thus, another application shows that the spectral p-norm and the p-spectral radius of a symmetric nonnegative r-matrix are equal whenever $p \geq r$. This result contributes to a classical area of analysis, initiated by Mazur and Orlicz back in 1930.

Additionally, a number of bounds are given on the p-spectral radius and the spectral p-norm of r-matrices and r-graphs.

Keywords: *spectral norm; hypermatrix; Schur's bound; p-spectral radius; nonnegative hypermatrix; hypergraph.*

AMS classification: *05C50, 05C65, 15A18, 15A42, 15A60, 15A69.*

1 Introduction

In this paper we study the spectral p-norm of hypermatrices and its applications to spectral hypergraph theory. Recall that the spectral 2-norm $\|A\|_2$ of an $m \times n$ matrix $A := [a_{i,j}]$ is defined as

$$\|A\|_2 := \max \ \{|\sum_{i,j} a_{i,j}x_jy_i| : \ |x_1|^2 + \cdots + |x_n|^2 = 1 \text{ and } \ |y_1|^2 + \cdots + |y_m|^2 = 1\}.$$

*Department of Mathematical Sciences, University of Memphis, Memphis TN 38152, USA. Email: *vnikifrv@memphis.edu*