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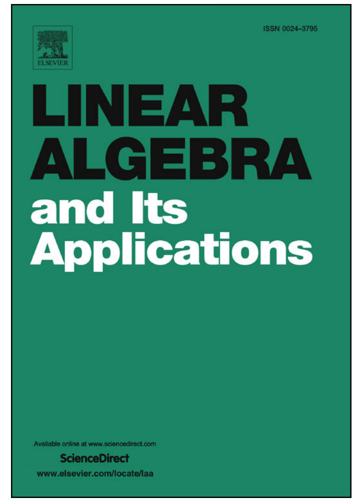

A unified perturbation analysis framework for countable Markov chains

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A unified perturbation analysis framework for countable Markov chains

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Abstract

In this paper, we are devoted to singular perturbation analysis for discrete-time or continuous-time Markov chains. We modify and extend the drift condition method, well known for regular perturbation, to develop a new framework for singular perturbation analysis. Our results extend and improve the corresponding ones in [2] for singularly perturbed Markov chains by allowing a general perturbation form, less restrictive conditions, and more computable bounds. Our analysis covers the regular perturbation analysis, and hence unifies singular and regular perturbation analysis. Furthermore, our results are illustrated by two two-dimensional Markov chains, including a discrete-time queue and a continuous-time level dependent quasi-birth-death process.

Keywords: Markov chains, queues, perturbation analysis, stationary distribution
AMS: 60J10, 60J27, 60J22

1 Introduction

Let $\widetilde{X}_n$ be a time-homogeneous discrete-time Markov chain (DTMC) on a countable state space $\mathbb{E}$ with an irreducible stochastic transition matrix $\widetilde{P} = (\widetilde{P}(i,j))$. The transition matrix $\widetilde{P}$ can be decomposed into

$$\widetilde{P} = P + \Delta. \tag{1.1}$$

In the context of perturbation, $\widetilde{P}$ is the perturbed transition matrix, P is the unperturbed transition matrix, and Δ is the perturbation matrix which is usually assumed small. The state space of the unperturbed transition matrix P consists of separate irreducible classes $\mathbb{E}_n$ for $n \in \hat{\mathbb{E}}$, where the set $\hat{\mathbb{E}} := \{0, 1, \cdots, \ell\}$, $0 \le \ell \le \infty$ is denumerable (finite or infinitely countable), and each class $\mathbb{E}_n$ is also denumerable. Note that $\tilde{P}$ and P have the same state space $\mathbb{E} = \bigcup_{n \in \hat{\mathbb{E}}} \hat{\mathbb{E}} \times \mathbb{E}_n$. The perturbation matrix Δ satisfies that $\Delta \boldsymbol{e} = \mathbf{0}$, where $\boldsymbol{e}$ is a column vector of ones, and $\mathbf{0}$ is a column vector of zeros. When $\ell = 0$, the perturbation is called regular perturbation, and when $\ell \ge 1$, the perturbation is called singular perturbation.

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