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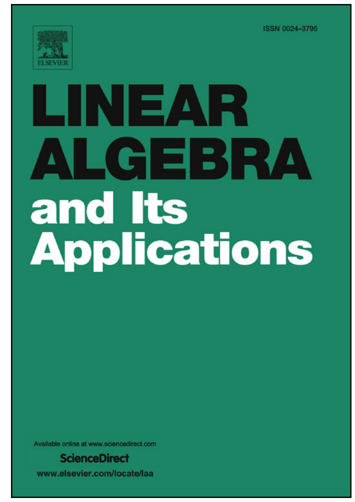

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CYCLIC TRIDIAGONAL PAIRS, HIGHER ORDER ONSAGER ALGEBRAS AND ORTHOGONAL POLYNOMIALS

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Abstract. The concept of cyclic tridiagonal pairs is introduced, and explicit examples are given. For a fairly general class of cyclic tridiagonal pairs with cyclicity N, we associate a pair of 'divided polynomials'. The properties of this pair generalize the ones of tridiagonal pairs of Racah type. The algebra generated by the pair of divided polynomials is identified as a higher-order generalization of the Onsager algebra. It can be viewed as a subalgebra of the $q-$Onsager algebra for a proper specialization at q the primitive $2N$th root of unity. Orthogonal polynomials beyond the Leonard duality are revisited in light of this framework. In particular, certain second-order Dunkl shift operators provide a realization of the divided polynomials at $N = 2$ or $q = \mathrm{i}$.

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Keywords: Tridiagonal pair; Tridiagonal algebra; $q-$Onsager algebra; Recurrence relations; Orthogonal polynomials; Leonard duality

Contents

1. Introduction

Let $\mathbb{K}$ denote a field. Let V denote a vector space over $\mathbb{K}$ with finite positive dimension. Recall that a *Leonard pair* is a pair of linear transformations A, A^* such that there exist two bases for V with respect to which the matrix representing A (resp. A^*) is irreducible tridiagonal (resp. diagonal) and the matrix representing A^* (resp. A) is diagonal (resp. irreducible tridiagonal) [T99]. For the well-known families of one-variable orthogonal polynomials in the Askey-scheme including the Bannai–Ito polynomials [BI84], the bispectral problem they solve finds a natural interpretation within the framework of Leonard's theorem [BI84] and Leonard pairs [T03]: given a Leonard pair, the overlap coefficients between the two bases coincide with polynomials of the Askey-scheme ($q-$Racah,