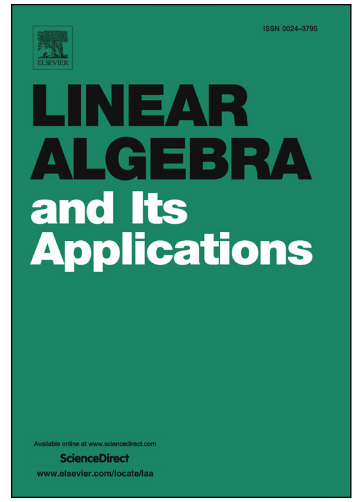
Accepted Manuscript

The real nonnegative inverse eigenvalue problem is NP-hard

Alberto Borobia, Roberto Canogar



PII: S0024-3795(17)30086-1
DOI: http://dx.doi.org/10.1016/j.laa.2017.02.010
Reference: LAA 14048

To appear in: *Linear Algebra and its Applications*

Received date: 29 July 2016
Accepted date: 6 February 2017

Please cite this article in press as: A. Borobia, R. Canogar, The real nonnegative inverse eigenvalue problem is NP-hard, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.02.010

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

The real nonnegative inverse eigenvalue problem is NP-hard *†‡

Alberto Borobia, Roberto Canogar

Dpto. Matemáticas, Universidad Nacional de Educación a Distancia (UNED), 28040 Madrid, Spain

e-mail: *aborobia@mat.uned.es*, *rcanogar@mat.uned.es*

Abstract

A list of complex numbers is realizable if it is the spectrum of a nonnegative matrix. In 1949 Suleĭmanova posed the nonnegative inverse eigenvalue problem (NIEP): the problem of determining which lists of complex numbers are realizable. The version for reals of the NIEP (RNIEP) asks for realizable lists of real numbers. In the literature there are many sufficient conditions or criteria for lists of real numbers to be realizable. We will present an unified account of these criteria. Then we will see that the decision problem associated to the RNIEP is NP-hard and we will study the complexity for the decision problems associated to known criteria.

1 Introduction

A matrix is *nonnegative* if all its entries are nonnegative numbers. The *Real Nonnegative Inverse Eigenvalue Problem* (which we will denote as **RNIEP**) asks for the characterization of all possible real spectra of nonnegative matrices. A list $\Lambda = (\lambda_1, \ldots, \lambda_n)$ of n real numbers is said to be *realizable* if there exists some nonnegative matrix $A \geq 0$ of order n with spectrum $\sigma(A) = \{\lambda_1, \ldots, \lambda_n\}$. With some abuse of notation, from now on we will use the expression $\sigma(A) = \Lambda$ or $\sigma(A) = (\lambda_1, \ldots, \lambda_n)$.

For $\Lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n$ define

$$\rho(\Lambda) = \max\{|\lambda_1|, \ldots, |\lambda_n|\} \quad \text{and} \quad \Sigma(\Lambda) = \lambda_1 + \cdots + \lambda_n.$$

We will restrict to lists of monotonically nonincreasing real numbers, that is, elements of the sets

$$\mathbb{R}^n_{\downarrow} \equiv \left\{(\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n : \lambda_1 \geq \cdots \geq \lambda_n\right\}.$$

If $\Lambda \in \mathbb{R}^n_{\downarrow}$ is the spectrum of a nonnegative matrix A then $\Sigma(\Lambda)$ is the trace of A (which implies that $\Sigma(\Lambda) \geq 0$) and $\rho(\Lambda)$ is the Perron eigenvalue of A (which implies that $\rho(\Lambda) = \lambda_1$). So the candidates to be a real spectrum of some nonnegative matrix belong to the set $\Pi_{\mathbb{R}} = \Pi^1_{\mathbb{R}} \cup \Pi^2_{\mathbb{R}} \cup \cdots$ where

$$\Pi^n_{\mathbb{R}} \equiv \left\{\Lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n_{\downarrow} : \Sigma(\Lambda) \geq 0;\ \rho(\Lambda) = \lambda_1\right\}.$$

The set of all real spectra of nonnegative matrices is $\Pi_{\text{RNIEP}} = \Pi^1_{\text{RNIEP}} \cup \Pi^2_{\text{RNIEP}} \cup \cdots$ where

$$\Pi^n_{\text{RNIEP}} = \{\Lambda \in \Pi^n_{\mathbb{R}} : \exists \text{ a nonnegative matrix } A \text{ of order } n \text{ with } \sigma(A) = \Lambda\}.$$

The RNIEP asks for the characterization of Π_{RNIEP}. The complete characterization of Π^n_{RNIEP} is only known for $n \leq 4$. Indeed this seems to be an intractable problem for large n. Nevertheless several subsets of Π_{RNIEP} are known. These partial solutions are presented in the literature as *criteria*, so that if $\Lambda \in \Pi_{\mathbb{R}}$

*Keywords: Nonnegative matrix, inverse eigenvalue problem, decision problem, P, NP, NP-hard, NP-complete.

†Mathematics subject classification: 15A18, 15A29, 90C60, 68Q25.

‡Supported by the Spanish Ministerio de Ciencia y Tecnología MTM2015-68805-REDT.

Download English Version:

https://daneshyari.com/en/article/5773291

Download Persian Version:

https://daneshyari.com/article/5773291

Daneshyari.com