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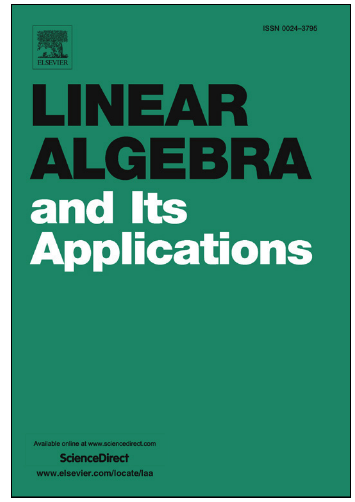


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# ON DIAMETER OF COMPONENTS IN COMMUTING GRAPHS

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Abstract. For each prime $p \geq 7$ we construct two matrices inside $M_{2p}(\mathbb{Q})$, such that their distance in a commuting graph $\Gamma(M_{2p}(\mathbb{Q}))$ equals six.

## 1. Introduction and Preliminaries

Commutativity is certainly one of the important algebraic relations; the whole sections of algebra are divided into Abelian and nonabelian ones. In the latter ones, one often investigates the commutativity and the lack of it with the help of a commutator of two elements, if the underlying algebraic structure is rich enough to allow defining it. A less informative tool is to utilize the centralizer of a given element $a$ in the algebra $A$ which we denote by $\mathcal{C}_A(a) := \{x \in A; \quad ax = xa\}$; its advantage over commutator is that the centralizer is well-defined in every algebraic structure. Recall that the center of $A$ is the set of all $a \in A$ with $\mathcal{C}_A(a) = A$.

The second approach is to investigate commutativity via graphs. This approach can be traced back at least as far as Brauer and Fowler [4], in their attempt towards classification of simple finite groups. More precisely, given an algebraic structure $A$, one lets $\Gamma = \Gamma(A)$ be a simple (= looples, undirected) graph, with vertex set equal to all noncentral elements from $A$, and where two distinct vertices form an edge if the corresponding elements commute in $A$. We remark that for abelian algebras, the corresponding commuting graph is empty (has no vertices).

Let us mention that commutativity relation alone sometimes suffices to determine the given algebraic structure up to isomorphism. Technically, one starts by assuming that the corresponding commuting graphs are isomorphic and then discerns the similarities among the corresponding algebras. For example, Solomon and Woldar [15] showed that if a commuting graph of nonabelian finite simple group $A$ is isomorphic to a commuting graph of some group $G$, then groups $A$ and $G$ are isomorphic. In a similar vein, inclined more towards our present results, Mohammadian [10] showed that a unital ring $R$ is isomorphic to a matrix algebra $M_2(\mathbb{F})$ of 2-by-2 matrices over a finite field $\mathbb{F}$ if and only if their commuting graphs $\Gamma(R)$ and $\Gamma(M_2(\mathbb{F}))$ are isomorphic (see also Abdollahi [1] for an earlier but slightly weaker result).

One of the basic questions with commuting graphs is their connectedness and their diameter. As far as $M_n(\mathbb{F})$, the algebra of $n$-by-$n$ matrices over a field $\mathbb{F}$ is concerned, which is the principal investigation topic of the present paper, it is known that its commuting

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