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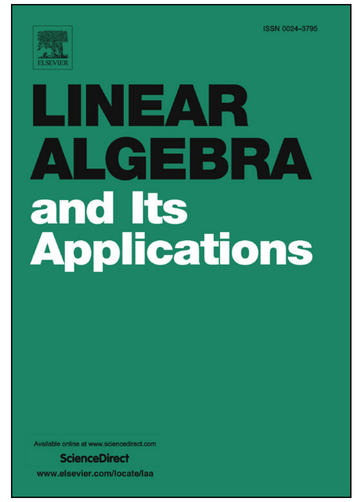

On equivalence of optimal relaxed block iterative methods for the singular nonsymmetric saddle point problem

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On equivalence of optimal relaxed block iterative methods for the singular nonsymmetric saddle point problem

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Abstract

There exist many classes of relaxed block iterative methods for the solution of the nonsingular and singular saddle point problems. Recently, the singular nonsymmetric saddle point problem has been optimally solved by means of a stationary linear second-order iterative method using the Manteuffel algorithm [A. Hadjidimos, The saddle point problem and the Manteuffel algorithm, BIT Numer. Math. 56 (2016) 1281–1302. The main purpose of this work is to extend, analyze and study a number of classes of stationary iterative methods based on generalizations of SOR-like methods, determine their optimal parameters, via the optimal parameters in the aforementioned work, and show the equivalence of the optimal methods studied. Finally, a computational comparison of the performances of the above optimal methods and their nonstationary counterparts shows the superiority of the latter methods.

AMS (MOS) Subject Classifications: Primary 65F10. Secondary 65F08, 65F35

Keywords: singular nonsymmetric saddle point problem, iterative methods, Manteuffel algorithm, optimal parameters, optimal semi-convergence factor

Running Title: Optimal methods for singular nonsymmetric saddle point problem

1 Introduction

The singular nonsymmetric saddle point problem is usually stated in the following form. Solve the linear system

$$\mathcal{A}\begin{bmatrix} x \\ y \end{bmatrix} := \begin{bmatrix} A & B \\ -B^T & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ -q \end{bmatrix}, \tag{1.1}$$

where $A \in \mathbb{R}^{m\times m}$ is nonsymmetric positive definite, $B \in \mathbb{R}^{m\times n}$, $n \le m$, is rank deficient, $\text{rank}(B) = r < n$, $p \in \mathbb{R}^m$, $q \in \mathbb{R}^n$, and the system (1.1) is consistent, i.e. $[p^T, -q^T]^T \in \text{range}(\mathcal{A})$.

Linear system (1.1) arises in various scientific and engineering applications. An account of the works before 2001 for the solution of (1.1) can be found in the work by Golub et al. [18]. This work was introduced to solve a particular case of (1.1) by the so called SOR-like method and was dedicated to the late Professor David M. Young (see, e.g., [35], [36], [37]) for his monumental contribution to the development of the classical SOR method. Also, an account of the works up to 2009, generalizing the idea of the SOR-like method [18], can be found in Zheng et al. [42]. Here are some of the main works in the area in the last sixteen years: Golub et al. [18], Bai et al. [5],