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Distance-regular graphs with small number of distinct distance eigenvalues

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ABSTRACT

In this paper we characterize distance-regular graphs with diameter three having exactly three distinct distance eigenvalues, and also bipartite distance-regular graphs with diameter four having three distinct distance eigenvalues. We derive some properties and give particular examples of such graphs. We also present an infinite family of bipartite semiregular graphs with diameter four having exactly four distinct distance eigenvalues. With these results, we address some problems posed in Atik and Panigrahi (2015) [3].

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1. Introduction

For a simple graph G of order $n = |V(G)|$, the characteristic polynomial P_G is defined as the characteristic polynomial of its adjacency matrix A $(= A(G))$. The *eigenvalues* of G,

$$\lambda_1(G) \geq \lambda_2(G) \geq \cdots \geq \lambda_n(G),$$

are then just the eigenvalues of A, and the *spectrum* of G is the multiset of its eigenvalues.

The *distance matrix* of a connected graph G is an $n \times n$ matrix $D = (d_{ij})$, where d_{ij} is the distance (length of a shortest path) between the vertices i and j. The *distance eigenvalues* (for short *D-eigenvalues*) of G are the eigenvalues of D. We denote them

$$\varrho_1(G) \geq \varrho_2(G) \geq \cdots \geq \varrho_n(G).$$

Similarly, D-eigenvalues form the multiset called the *D-spectrum* of G.

In this paper we consider graphs with three or four distinct D-eigenvalues. In the major part we restrict ourselves to the distance-regular graphs (the definition is given in the next section), and so we characterize distance-regular graphs with diameter three having exactly three distinct D-eigenvalues. In this context, we give a closer description of bipartite distance-regular graphs with the same properties. Further on, we also characterize bipartite distance-regular graphs with diameter four and exactly four distinct D-eigenvalues. At the end, we present an infinite family of bipartite semiregular graphs with diameter four and exactly four distinct D-eigenvalues. Some recent results considering similar topics can be found in [1,2].

In [3] the authors gave a number of examples of distance-regular graphs with diameter d and less than $d + 1$ distinct D-eigenvalues. In the same paper some problems were posed. The first is a problem of characterizing distance-regular graphs having exactly three distinct D-eigenvalues. The second is a problem of characterizing distance-regular graphs with diameter d having exactly $d + 1$ distinct D-eigenvalues, while the third is a question on existence of graphs other than distance-regular with diameter d and strictly less than $d + 1$ distinct D-eigenvalues.

Here we partially address the first two problems by considering specified distance-regular graphs with diameter three or four. We also give an affirmative answer to the last question.

The paper is organized as follows. In Section 2 we introduce terminology and notation, and recall on some necessary results. In Section 3 we deal with distance-regular graphs with diameter three having also three distinct D-eigenvalues. In Section 4 we increase the diameter to four. An infinite family of non-regular graphs with diameter four having also four distinct D-eigenvalues is derived in Section 5.