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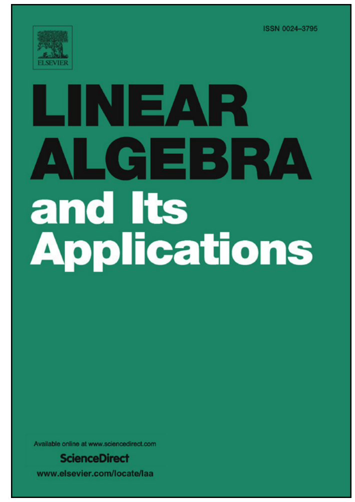

Some properties of the Laplace and normalized Laplace spectra of uniform hypergraphs

Jiayu Shao, Xiying Yuan

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Some properties of the Laplace and normalized Laplace spectra of uniform hypergraphs *

Jiayu Shao † Xiying Yuan‡§

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Abstract

In [S. Hu, L. Qi, The Laplacian of a uniform hypergraph, Journal of Combinatorial Optimization, 29(2015),331-366.], Hu and Qi studied the normalized Laplace tensors and normalized Laplace spectra of k-uniform hypergraphs. They also mentioned the question about whether or not 2 is also an H-eigenvalue of the normalized Laplace tensor of a k-uniform hypergraph, when 2 is an eigenvalue of the normalized Laplace tensor (in this case, k is necessarily even).

In this paper, we use an expression for the normalized Laplace tensor in terms of the tensor product, together with the diagonal similarity of tensors, the Perron-Frobenius Theorem for nonnegative tensors and nonnegative weakly irreducible tensors, and the concept and properties of odd-colorable hypergraphs introduced in [V. Nikiforov, Hypergraphs and hypermatrices with symmetric spectrum, Linear Algebra Appl.,519 (2017)1-18.], to give a complete answer to this question. We show that: (i). When $k \equiv 2(mod\ 4)$, then the answer to this question is affirmative. (ii). When $k \equiv 0(mod\ 4)$, then the answer to this question is negative, and in this case, we give an infinite family of counterexamples.

We also study the signless normalized Laplace spectra and the signless normalized Laplace H-spectra of hypergraphs. We give structural characterizations of the hypergraphs having the same normalized Laplace spectrum and signless normalized Laplace spectrum, or having the same normalized Laplace H-spectrum and signless normalized Laplace H-spectrum, or both. Finally, we determine the first two k-uniform supertrees of order n with the largest Laplace spectral radii, and also determine the unique k-uniform hypertree of order n with the smallest Laplace spectral radii, in the case when k is even.

AMS classification: *15A42, 05C50*

Keywords: *Uniform hypergraph; normalized Laplace tensor; signless normalized Laplace tensor; odd-colorable; odd-bipartite; supertree; spectral radius; H-eigenvalue*

1 Introduction

In 1997, Chung ([3]) first introduced the concept of the normalized Laplace matrix and its spectrum of an ordinary graph. Let G be a graph of order n with no isolated vertices, L be the Laplace matrix of G. Write $D = diag(d_1, \cdots, d_n)$ to be the degree diagonal matrix of G, where $d_i = d(v_i)$ is the degree of the vertex v_i of G. Then the matrix $D^{-\frac{1}{2}} L D^{-\frac{1}{2}}$ is called the normalized Laplace matrix of G, denoted by $N(G)$.

It is not difficult to see from the definition that, by the congruence relation of the symmetric matrices and the positive semi-definiteness of L, the normalized Laplace matrix $N(G)$ is also positive semi-definite.

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†School of Mathematical Sciences, Tongji University, Shanghai, China, *Email address: jyshao@tongji.edu.cn*

‡Corresponding author

§Department of Mathematics, Shanghai University, Shanghai 200444, China; *E-mail address: xiyingyuan2007@hotmail.com*