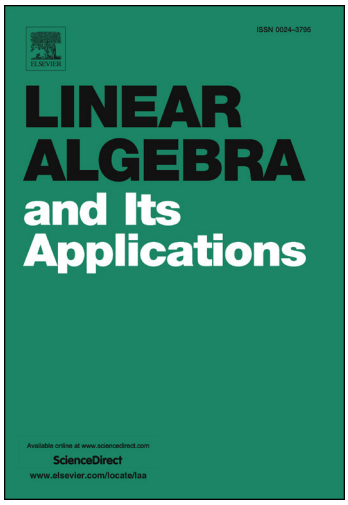

On the algorithm by Al-Mohy and Higham for computing the action of the matrix exponential: A posteriori roundoff error estimation

Thomas M. Fischer

PII: S0024-3795(17)30342-7
DOI: http://dx.doi.org/10.1016/j.laa.2017.05.042
Reference: LAA 14190

To appear in: *Linear Algebra and its Applications*

Received date: 23 September 2016
Accepted date: 25 May 2017

Please cite this article in press as: T.M. Fischer, On the algorithm by Al-Mohy and Higham for computing the action of the matrix exponential: A posteriori roundoff error estimation, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.05.042

On the algorithm by Al-Mohy and Higham for computing the action of the matrix exponential: A posteriori roundoff error estimation

Thomas M. Fischer

Department of Mathematics and Natural Sciences, University of Applied Sciences, D-64295 Darmstadt, Germany

Abstract

The algorithm by Al-Mohy and Higham (2011) computes an approximation to $e^A b$ for given A and b, where A is an n-by-n matrix and b is, for example, a vector of dimension n. It uses a scaling together with a truncated Taylor series approximation to the exponential of the scaled matrix. In this paper, a method is developed for estimating the roundoff error of the computed solution. An asymptotic expansion of this error for small values of the unit roundoff is the basis of the method. The roundoff error is further expressed in terms of sums of rounding errors, which occur during the computation. A second approximation to $e^A b$, which is computed with a lower precision than the first one, is used to evaluate these rounding errors. The result is an upper bound on the normwise relative roundoff error. Further, an algorithm is proposed for computing the error bound. The cost for performing this algorithm depends on the type of problem and the accuracy, which is required for the error estimate. In case that all computations are performed with standard precisions, this cost can be expressed in terms of the number of computed matrix-vector products and is bounded from above by two times the cost for computing $e^A b$.

Keywords: matrix exponential, Taylor series, roundoff error, asymptotic expansion, a posteriori error estimate
2010 MSC: 65F60

Email address: thomas.fischer@h-da.de (Thomas M. Fischer)