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A new characterization of simultaneous Lyapunov diagonal stability via Hadamard products

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ABSTRACT

A well-known characterization by Kraaijevanger [14] for Lyapunov diagonal stability states that a real, square matrix A is Lyapunov diagonally stable if and only if $A \circ S$ is a P-matrix for any positive semidefinite S with nonzero diagonal entries. This result is extended here to a new characterization involving similar Hadamard multiplications for simultaneous Lyapunov diagonal stability on a set of matrices. Among the main ingredients for this extension are a new concept called $\mathcal{P}$-sets and a recent result regarding simultaneous Lyapunov diagonal stability by Berman, Goldberg, and Shorten [2].

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1. Introduction

In this paper, we shall deal only with real and square matrices. The size of a matrix may not be specified if it is clear from the context. For convenience, the notion of positive stability shall be adopted; that is, a matrix A is said to be stable if each eigenvalue of A has a positive real part. By the classical Lyapunov's theorem, see, for example, [12, Theorem 2.2.1], A being stable is equivalent to the existence of a (symmetric) positive definite matrix P such that

$$AP + PA^T \tag{1}$$

is positive definite. From now on, for brevity, we shall denote a positive definite (resp. semidefinite) matrix X by $X \succ 0$ (resp. $X \succeq 0$).

The notion of so-called Lyapunov diagonal stability arises when the matrix P as in (1) has a diagonal structure. Formally, a matrix A is said to be Lyapunov diagonally stable if there exists a diagonal matrix $D \succ 0$ so that

$$AD + DA^T \succ 0. \tag{2}$$

Such a matrix D is called in the literature a DLS, which stands for diagonal Lyapunov solution, to (2) or a DLS for the matrix A. In light of Lyapunov's theorem, Lyapunov diagonal stability is clearly a stronger condition as compared with the regular stability.

Lyapunov diagonal stability poses a very interesting research problem of its own right, see the survey in [9] for relevant results including the connection between Lyapunov diagonal stability and various other types of matrix stability. Meanwhile, it plays an important role in applied problems such as population dynamics [10], systems theory [13], and complex networks [16].

In what follows, for cleaner notation of entries, each element in a general matrix or vector set shall be identified by a superscript.

Recently, an extension of the above DLS problem has drawn much attention as well, see [4,8,15,18,22,23]. Specifically, let $\mathcal{A} = \{A^{(1)}, A^{(2)}, \ldots, A^{(m)}\}$ be a set of matrices of the same size, then this extension concerns whether there exists a diagonal matrix $D \succ 0$ such that

$$A^{(k)}D + D(A^{(k)})^T \succ 0 \tag{3}$$

for all $k = 1, 2, \ldots, m$. Such a matrix D, when it exists, is called a common diagonal Lyapunov solution, or CDLS for short in the rest of this paper, to (3). We shall also refer in the sequel to this matrix D simply as a CDLS for the matrix set $\mathcal{A}$. Clearly, the existence of a CDLS for $\mathcal{A}$ translates to the so-called simultaneous Lyapunov diagonal stability of all the matrices in $\mathcal{A}$. Incidentally, a CDLS is just a DLS when $\mathcal{A}$ is a singleton.

The motivation behind the CDLS problem is partly attributed to a result of Redheffer [19], re-examined in [8,23], which reduces the DLS problem on an $n \times n$ matrix A to the