Accepted Manuscript

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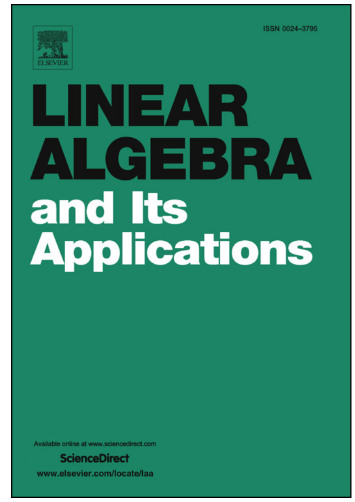

PII: S0024-3795(17)30363-4
DOI: http://dx.doi.org/10.1016/j.laa.2017.06.001
Reference: LAA 14199

To appear in: *Linear Algebra and its Applications*

Received date: 1 March 2017
Accepted date: 1 June 2017

Please cite this article in press as: J. Chmieliński et al., Approximate orthogonality in normed spaces and its applications, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2017.06.001

Approximate orthogonality in normed spaces and its applications

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Abstract

In a normed space we consider an approximate orthogonality relation related to the Birkhoff orthogonality. We give some properties of this relation as well as applications. In particular, we characterize the approximate orthogonality in the class of linear bounded operators on a Hilbert space.

Keywords: Birkhoff orthogonality; approximate Birkhoff orthogonality; approximate orthogonality of operators
2010 MSC: 46B20, 46B28, 46C05, 46C50, 47L25, 47L30

1. Introduction

In an inner product space $(X, \langle\cdot|\cdot\rangle)$, with the standard orthogonality relation $x \perp y \Leftrightarrow \langle x|y\rangle = 0$, a natural way to define an *approximate orthogonality* (or more precisely ε-*orthogonality* with $\varepsilon \in [0,1)$) is by:

$$x \perp^{\varepsilon} y \iff |\langle x|y\rangle| \leq \varepsilon \|x\| \, \|y\|, \qquad x, y \in X.$$

It is easy to show, in this setting, the following characterization:

$$x \perp^{\varepsilon} y \iff \exists\, z \in X: \; x \perp z, \; \|z - y\| \leq \varepsilon \|y\|. \tag{1.1}$$

Indeed, if $x \perp^{\varepsilon} y$, then it is enough to take $z = -\frac{\langle x|y\rangle}{\|x\|^2} x + y$ for $x \neq 0$ and $z = y$ for $x = 0$. Conversely, assuming $x \perp z$ and $\|z - y\| \leq \varepsilon \|y\|$, we get $|\langle x|y\rangle| = |\langle x|y - z\rangle| \leq \|x\| \, \|y - z\| \leq \varepsilon \|x\| \, \|y\|$, that is $x \perp^{\varepsilon} y$. Notice that z

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