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The asymptotic analysis of the structure-preserving doubling algorithms

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ABSTRACT

This paper is the second part of [15]. Taking advantage of the special structure and properties of the Hamiltonian matrix, we apply a symplectically similar transformation introduced by [18] to reduce $\mathscr{H}$ to a Hamiltonian Jordan canonical form $\mathfrak{J}$. The asymptotic analysis of the structure-preserving flows and RDEs is studied by using $e^{\mathfrak{J}t}$. The convergence of the SDA as well as its rate can thus result from the study of the structure-preserving flows. A complete asymptotic dynamics of the SDA is investigated, including the linear and quadratic convergence studied in the literature [3,12,13].

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1. Introduction

There are two important types of nonlinear matrix equations arising from several practically important applications. One type is algebraic Riccati equations which have two subtypes, namely, Discrete-time Algebraic Riccati Equation (DARE) and Continuous-time Algebraic Riccati Equation (CARE), respectively [16,20],

$$X = A^H X(I + GX)^{-1}A + H,$$
$$-XGX + A^H X + XA + H = 0,$$

where A, $G^H = G$, $H^H = H \in \mathbb{C}^{n\times n}$. The other type is Nonlinear Matrix Equation (NMEs) [7] in the form

$$X + A^H X^{-1} A = Q,$$

where A, $Q^H = Q \in \mathbb{C}^{n\times n}$.

The Structure-Preserving Doubling Algorithms (SDAs) [3,13,19] are usually employed for solving the stabilizing solutions of DAREs, CAREs and NMEs. For solving DAREs, the symplectic pairs $(\mathcal{M}_k, \mathcal{L}_k) = \left(\begin{bmatrix} A_k & 0 \\ -H_k & I \end{bmatrix}, \begin{bmatrix} I & G_k \\ 0 & A_k^H \end{bmatrix}\right)$ are generated by

Algorithm SDA-1 [13,19].

Let $A_1 = A$, $G_1 = G$, and $H_1 = H$. For $k = 1, 2, \ldots,$ compute

$$A_{k+1} = A_k(I + G_k H_k)^{-1} A_k,$$
$$G_{k+1} = G_k + A_k G_k(I + H_k G_k)^{-1} A_k^H,$$
$$H_{k+1} = H_k + A_k^H (I + H_k G_k)^{-1} H_k A_k.$$

For solving CAREs, one can transform it into a DARE by using a suitable Cayley transformation [21]. Then **Algorithm** SDA-1 can be employed to find the desired stabilizing solution of CAREs. For solving NMEs, the symplectic pairs $(\mathcal{M}_k, \mathcal{L}_k) = \left(\begin{bmatrix} A_k & 0 \\ Q_k & -I \end{bmatrix}, \begin{bmatrix} -B_k & I \\ A_k^H & 0 \end{bmatrix}\right)$ are generated by

Algorithm SDA-2 [3,19].

Let $A_1 = A$, $Q_1 = Q$, and $B_1 = 0$. For $k = 1, 2, \ldots,$ compute

$$A_{k+1} = A_k(Q_k - P_k)^{-1} A_k,$$
$$Q_{k+1} = Q_k - A_k^H (Q_k - B_k)^{-1} A_k,$$
$$B_{k+1} = B_k + A_k(Q_k - B_k)^{-1} A_k^H.$$