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Classification of linear mappings between indefinite inner product spaces

Juan Meleiro [a], Vladimir V. Sergeichuk [b,*], Thiago Solovera [a], André Zaidan [a]

[a] Instituto de Matemática e Estatística, Universidade de São Paulo, Brazil
[b] Institute of Mathematics, Tereshchenkivska 3, Kiev, Ukraine

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ABSTRACT

Let $\mathcal{A} : U \to V$ be a linear mapping between vector spaces U and V over a field or skew field $\mathbb{F}$ with symmetric, or skew-symmetric, or Hermitian forms $\mathcal{B} : U \times U \to \mathbb{F}$ and $\mathcal{C} : V \times V \to \mathbb{F}$.
We classify the triples $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ if $\mathbb{F}$ is $\mathbb{R}$, or $\mathbb{C}$, or the skew field of quaternions $\mathbb{H}$. We also classify the triples $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ up to classification of symmetric forms and Hermitian forms if the characteristic of $\mathbb{F}$ is not 2.

* Corresponding author.
E-mail addresses: juan.meleiro@me.com (J. Meleiro), sergeich@imath.kiev.ua (V.V. Sergeichuk), thiago.solovera.nery@usp.br (T. Solovera), andre.zaidan@gmail.com (A. Zaidan).

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1. Introduction

We consider a triple

$$\mathcal{A} : U \to V, \qquad \mathcal{B} : U \times U \to \mathbb{F}, \qquad \mathcal{C} : V \times V \to \mathbb{F} \tag{1}$$

consisting of a linear mapping $\mathcal{A}$ and two forms $\mathcal{B}$ and $\mathcal{C}$ on finite-dimensional vector spaces U and V over a field or skew field $\mathbb{F}$ of characteristic not 2. Each of the forms $\mathcal{B}$ and $\mathcal{C}$ is either symmetric or skew-symmetric if $\mathbb{F}$ is a field, or both the forms are Hermitian with respect to a fixed nonidentity involution in $\mathbb{F}$.

A canonical form of the triple of matrices of (1) over a field $\mathbb{F}$ of characteristic not 2 was obtained in the deposited manuscript [22] up to classification of Hermitian forms over finite extensions of $\mathbb{F}$. The aim of this paper is to give a detailed exposition of this result and extend it to triples (1) over a skew field of characteristic not 2. We give canonical matrices of (1) over $\mathbb{R}$, $\mathbb{C}$, and the skew field of quaternions $\mathbb{H}$.

Other canonical matrices of (1) with nonsingular forms $\mathcal{B}$ and $\mathcal{C}$ over the fields $\mathbb{R}$ and $\mathbb{C}$ were given by Mehl, Mehrmann, and Xu [14–16], and by Bolshakov and Reichstein [2].

Following [22], we represent the triple (1) by the graph

$$\begin{tikzcd} U \arrow[d, "\mathcal{A}"'] \arrow[loop right, "\varepsilon"] & \mathcal{B} \\ V \arrow[loop right, "\delta"] & \mathcal{C} \end{tikzcd} \tag{2}$$

in which $\varepsilon = +$ if $\mathcal{B}$ is symmetric or Hermitian and $\varepsilon = -$ if $\mathcal{B}$ is skew-symmetric; $\delta = +$ if $\mathcal{C}$ is symmetric or Hermitian and $\delta = -$ if $\mathcal{C}$ is skew-symmetric.

Choosing bases in U and V, we give (1) by the triple (A, B, C) of matrices of $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$. Changing bases, we can reduce it by transformations

$$(A, B, C) \mapsto (S^{-1}AR, R^{\star}BR,\ S^{\star}CS), \tag{3}$$

in which R and S are nonsingular and

$$M^{\star} = M^{\top} \quad \text{or} \quad M^{\star} = \widetilde{M}^{\top}$$

with respect to a fixed involution $a \mapsto \tilde{a}$ in $\mathbb{F}$. Thus, we consider the canonical form problem for matrix triples under transformations (3). We represent the matrix triple (A, B, C) by the graph

$$\begin{tikzcd} m \arrow[d, "A"'] \arrow[loop right, "\varepsilon"] & B & m := \dim U, \\ n \arrow[loop right, "\delta"] & C & n := \dim V. \end{tikzcd} \tag{4}$$