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Positive linear maps and perturbation bounds of matrices

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ABSTRACT

We show how positive unital linear maps can be used to obtain lower bounds for the maximum distance between the eigenvalues of two normal matrices. Some related bounds for the spread and condition number of Hermitian matrices are also discussed here.

1. Introduction

Recently, Bhatia and Sharma [4–6] have shown that how positive unital linear maps can be used to obtain matrix inequalities. In particular, they have obtained some old and new lower bounds for the spread of a matrix. In this paper we show that their technique

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can be extended and positive unital linear maps can also be used to study the spectral variations of Hermitian and normal matrices.

Let $\mathbb{M}(n)$ be the algebra of all $n \times n$ complex matrices. Let $\langle x, y\rangle$ be the standard inner product on C^n defined as $\langle x, y\rangle = \sum_{i=1}^{n} \overline{x_i} y_i$, and let $\|x\| = \langle x, x\rangle^{\frac{1}{2}}$. The numerical range of an element $A \in \mathbb{M}(n)$ is the set

$$W(A) = \{\langle x, Ax\rangle : \|x\| = 1\}.$$

The Toeplitz–Hausdorff Theorem [7,12] says that $W(A)$ is a convex subset of the complex plane for all $A \in \mathbb{M}(n)$. For a normal matrix A,

$$W(A) = Co(\sigma(A))$$

where $Co(\sigma(A))$ denotes the convex hull of the spectrum $\sigma(A)$ of A. For non-normal matrices, $W(A)$ may be bigger than $Co(\sigma(A))$. The diameter of $W(A)$ is defined as

$$\text{diam } W(A) = \max_{i,j} \{|z_i - z_j| : z_i, z_j \in W(A)\}.$$

A linear map $\Phi : \mathbb{M}(n) \longrightarrow \mathbb{M}(k)$ is called positive if $\Phi(A)$ is positive semidefinite (psd) whenever A has that property, and unital if $\Phi(I) = I$. When $k = 1$, such a map is called positive, unital, linear functional and is denoted by the lower case letter φ.

Bhatia and Davis [3] have proved that if Φ is any positive unital linear map and the spectrum of any Hermitian matrix A is contained in the interval $[m, M]$, then

$$\Phi(A^2) - \Phi(A)^2 \leq \frac{(M-m)^2}{4} I. \tag{1.1}$$

Bhatia and Sharma [4] have extended this for arbitrary matrices. One more extension of (1.1) in the special case when A is normal and φ is linear functional is given in [6]. They have augmented this technique with another use of positive unital linear maps and showed that if Φ_1 and Φ_2 are positive unital linear maps from $\mathbb{M}(n)$ to $\mathbb{M}(k)$, then for every Hermitian matrix $A \in \mathbb{M}(n)$ we have

$$\|\Phi_1(A) - \Phi_2(A)\| \leq \text{diam } W(A) \tag{1.2}$$

where $\|\cdot\|$ denotes the spectral norm. Further, if φ_1 and φ_2 are positive unital linear functionals on $\mathbb{M}(n)$, then for every matrix A in $\mathbb{M}(n)$

$$|\varphi_1(A) - \varphi_2(A)| \leq \text{diam } W(A). \tag{1.3}$$

For more details, see [5,6]. Using these inequalities they have derived various old and new bounds for the spread of matrices. In a similar spirit we discuss here perturbation bounds related to inequalities involving positive linear maps.

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