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A SYMMETRIZATION OF THE JORDAN CANONICAL FORM

M. RADJABALIPOUR

ABSTRACT. For a (finite or infinite dimensional) vector space V , the notion of a symmetric Jordan canonical form of an operator $T \in L(V)$ having a minimal polynomial is defined and used to verify the relation between the notions of "Jordan canonical form" and "rational canonical form." The paper extends and repairs Theorem 2.2 of [M. Radjabalipour, *The rational canonical form via the splitting field*; Linear Algebra and Its Applications, 439 (2013), no. 8, 2250-2255]. In particular, it is shown that there exists an auxiliary nilpotent operator $S \in L(W)$, depending on T , such that every Jordan canonical form of S yields a symmetric Jordan canonical form and, if the characteristic of the ground field is zero, a rational canonical form for T . The paper concludes with a direct proof of the symmetric Jordan canonical form which "integrates" into a rational canonical form.

Dedicated to Rajendra Bhatia

1. INTRODUCTION

Let T be a linear operator on a finite or infinite dimensional vector space V over a general field \mathbb{F} . For $v \in V$, let $Z(v, T)$ denote the cyclic invariant subspace of T spanned by $\{v, Tv, T^2v, \dots\}$ (in the domain of T). The generator $f(x) := x^k + a_{k-1}x^{k-1} + \dots + a_1x + a_0$ of the principal ideal domain consisting of all polynomials $g \in \mathbb{F}[x]$ satisfying $g(T)v = 0$ is called the minimal T -annihilator of v . The space $Z(v, T)$ is of finite dimension k if and only if the minimal T -annihilator of v has degree k . If $v_1, v_2, \dots, v_k \in V$ have relatively prime minimal T -annihilators g_1, g_2, \dots, g_k , then $v = v_1 + v_2 + \dots + v_k$ has a minimal T -annihilator $h := h_1 h_2 \cdots h_k$ with $h_i | g_i \forall i$; since $(h_1 g_2 g_3 \cdots g_k)(T)v_1 = 0$, it follows that $g_1 | h_1$ and, thus, $g_1 = h_1$. By symmetry, $g_i = h_i \forall i$ and, in view of the primary decomposition theorem, there exist $w_i \in Z(v, T)$ such that $v = w_1 + w_2 + \dots + w_k$ and w_i has a minimal T -annihilator ($i = 1, 2, \dots, k$) and

$$Z(v, T) = Z(w_1, T) \oplus Z(w_2, T) \oplus \cdots \oplus Z(w_k, T).$$

Now, $(g_2 g_3 \cdots g_k)(T)(v_1 - w_1) = 0$ which implies that $g_1 | g_2 g_3 \cdots g_k$ if $v_1 \neq w_1$; the former being impossible, we conclude that $v_1 = w_1$ and, by symmetry,

$$(1.1) \quad Z(v, T) = Z(v_1, T) \oplus Z(v_2, T) \oplus \cdots \oplus Z(v_k, T).$$

The rational canonical form of T together with the corresponding cyclic decomposition of V assert that if V is finite dimensional [3, 4, 9] or, more generally, if T

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