

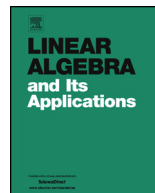


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## Linear Algebra and its Applications

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A class of simple matrix algebras <sup>☆</sup>Antonio J. Calderón Martín <sup>\*</sup>, Francisco J. Navarro Izquierdo

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## ABSTRACT

Fix an infinite set  $I$  and consider the associative matrix algebra  $\mathcal{M}_I(\mathbb{F})$  where  $\mathbb{F}$  is a base field with  $\text{char}(\mathbb{F}) \neq 2$ . For any couple of bijective maps  $\sigma, \nu : I \rightarrow I$ , such that  $\sigma\nu = \nu\sigma$  and  $\sigma^2 = \nu^2$ , we introduce a linear subspace  $\Omega(\sigma, \nu)$  of  $\mathcal{M}_I(\mathbb{F})$ . We endow it with a structure of (non-associative) algebra for a certain bilinear product, and obtain a wide class of non-associative algebras containing, in particular, the Lie algebras  $\text{Lie}(\mathcal{M}_I(\mathbb{F}), t)$ . We show that each algebra  $\Omega(\sigma, \nu)$  is simple.

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## 1. Introduction and previous definitions

Fix a base field  $\mathbb{F}$  with  $\text{char}(\mathbb{F}) \neq 2$ , an infinite set of indexes  $I$ , and the linear space of  $(I \times I)$ -matrices over  $\mathbb{F}$ , denoted by  $\mathcal{M}_I(\mathbb{F})$ . That is,  $\mathcal{M}_I(\mathbb{F})$  is the set of mappings  $I \times I \rightarrow \mathbb{F}$  with just a finite number of non-zero values. For any  $(i, j) \in I \times I$ , we will denote by  $E_{i,j}$  the elementary matrix given by  $E_{i,j}(i, j) = 1$  and  $E_{i,j}(p, q) = 0$  if  $(p, q) \neq (i, j)$ .

For any bijective map  $\sigma : I \rightarrow I$  we will denote by

$$F_\sigma : \mathcal{M}_I(\mathbb{F}) \rightarrow \mathcal{M}_I(\mathbb{F})$$

the linear bijection determined by  $F_\sigma(E_{i,j}) = E_{\sigma(i),\sigma(j)}$ , which is an associative automorphism of  $\mathcal{M}_I(\mathbb{F})$ . Also, for any bijective map  $\nu : I \rightarrow I$  we will denote by

$$G_\nu : \mathcal{M}_I(\mathbb{F}) \rightarrow \mathcal{M}_I(\mathbb{F})$$

the linear bijection defined by  $G_\nu(E_{i,j}) = E_{\nu(j),\nu(i)}$ , in which  $G_\nu$  is an associative anti-automorphism of  $\mathcal{M}_I(\mathbb{F})$ . That is,  $G_\nu(AB) = G_\nu(B)G_\nu(A)$  for any  $A, B \in \mathcal{M}_I(\mathbb{F})$ .

Fix now a couple of bijective maps  $\sigma, \nu : I \rightarrow I$  and consider the associative automorphism  $F_\sigma$  and the associative anti-automorphism  $G_\nu$ , constructed as above. Let us define the linear subspace of  $\mathcal{M}_I(\mathbb{F})$  given by

$$\Omega(\sigma, \nu) := \{M \in \mathcal{M}_I(\mathbb{F}) : F_\sigma(M) = -G_\nu(M)\}.$$

**Example 1.1.** If  $\sigma = \nu = Id$ , then  $F_\sigma$  is the identity on  $\mathcal{M}_I(\mathbb{F})$  while  $G_\nu$  is the transposition map. Hence,  $\Omega(Id, Id) = \text{Lie}(\mathcal{M}_I(\mathbb{F}), t)$ .

We note that if  $I = \mathbb{N}$ , then we obtain a known countably dimensional simple orthogonal Lie algebra  $\mathfrak{so}_\infty(\mathbb{F})$  which is a direct limit of finite dimensional simple orthogonal Lie algebras  $\mathfrak{so}(n, \mathbb{F})$  under natural embeddings; see [2].

**Example 1.2.** Let us fix  $I = \mathbb{C}$  the set of complex numbers,  $\mathbb{F} = \mathbb{R}$  the field of real numbers, and consider the linear space  $\mathcal{M}_\mathbb{C}(\mathbb{R})$ . If we fix  $\sigma, \nu : \mathbb{C} \rightarrow \mathbb{C}$  as  $\sigma(z) = \bar{z}$ , the conjugation map, and  $\nu = Id$ , we can consider the linear space

$$\Omega(\sigma, Id) = \{M \in \mathcal{M}_\mathbb{C}(\mathbb{R}) : F_\sigma(M) = -M^t\}.$$

Suppose now that  $\sigma\nu = \nu\sigma$  and  $\sigma^2 = \nu^2$ . Then we can introduce the bilinear product

$$\langle \cdot, \cdot \rangle : \Omega(\sigma, \nu) \times \Omega(\sigma, \nu) \rightarrow \Omega(\sigma, \nu)$$

on the linear subspace  $\Omega(\sigma, \nu)$  as follows:

$$\langle A, B \rangle := F_\sigma(AB) - G_\nu(AB)$$

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