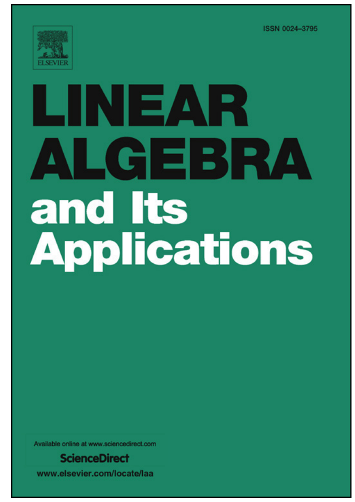
A Necessary Condition for the Spectrum of Nonnegative Symmetric 5×5 Matrices

Raphael Loewy, Oren Spector



PII: S0024-3795(16)30333-0
DOI: http://dx.doi.org/10.1016/j.laa.2016.08.006
Reference: LAA 13812

To appear in: *Linear Algebra and its Applications*

Received date: 6 April 2016
Accepted date: 3 August 2016

Please cite this article in press as: R. Loewy, O. Spector, A Necessary Condition for the Spectrum of Nonnegative Symmetric 5×5 Matrices, *Linear Algebra Appl.* (2016), http://dx.doi.org/10.1016/j.laa.2016.08.006

A Necessary Condition for the Spectrum of Nonnegative Symmetric 5×5 Matrices

Raphael Loewy[a,*], Oren Spector[a]

[a]Department of Mathematics, Technion - Israel Institute of Technology, Haifa 32000, Israel

Abstract

Let A be a nonnegative symmetric 5×5 matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq \lambda_5$. We show that if $\sum_{i=1}^{5} \lambda_i \geq \frac{1}{2}\lambda_1$ then $\lambda_3 \leq \sum_{i=1}^{5} \lambda_i$. McDonald and Neumann showed that $\lambda_1 + \lambda_3 + \lambda_4 \geq 0$. Let $\sigma = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ be a list of decreasing real numbers satisfying:

1. $\sum_{i=1}^{5} \lambda_i \geq \frac{1}{2}\lambda_1$,
2. $\lambda_3 \leq \sum_{i=1}^{5} \lambda_i$,
3. $\lambda_1 + \lambda_3 + \lambda_4 \geq 0$,
4. the Perron property, that is $\lambda_1 = \max_{\lambda \in \sigma} |\lambda|$.

We show that σ is the spectrum of a nonnegative symmetric 5×5 matrix. Thus, we solve the symmetric nonnegative inverse eigenvalue problem for $n = 5$ in a region for which a solution has not been known before.

Keywords: Symmetric nonnegative inverse eigenvalue problem, Extreme matrix
2010 MSC: 15A29, 15B48

1. Introduction

Let $\sigma = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ be a list of complex numbers. The problem of determining when σ is the spectrum of a nonnegative $n \times n$ matrix is called the nonnegative inverse eigenvalue problem (NIEP). If σ consists of real numbers, the problem of determining when σ is the spectrum of a nonnegative (symmetric nonnegative) $n \times n$ matrix is called RNIEP (SNIEP). All three problems are currently unsolved in the general case, more precisely for any list consisting of $n \geq 5$ numbers.

Loewy and London [6] have solved NIEP in the case of 3×3 matrices and RNIEP in the case of 4×4 matrices. Moreover, RNIEP and SNIEP are the same in the case of $n \times n$ matrices for $n \leq 4$. This can be seen from [6] and a paper by Fiedler [2]. However, it has been shown by Johnson, Laffey and Loewy [4] that RNIEP and SNIEP are different in general.

*Corresponding author
Email addresses: loewy@techunix.technion.ac.il (Raphael Loewy), spectoro@techunix.technion.ac.il (Oren Spector)