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## SOME INEQUALITIES FOR THE MATRIX HERON MEAN

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ABSTRACT. In this paper, we prove some norm inequalities for the matrix Heron mean of two positive definite matrices. We also prove a determinant inequality for the  $t$ -power mean of two positive definite matrices. As a consequence, we obtain a determinant inequality for the matrix Heron mean.

## 1. INTRODUCTION

Let  $M_n$  be the space of  $n \times n$  complex matrices and  $M_n^+$  the cone of positive semidefinite matrices. Denote by  $I$  the identity element of  $M_n$ . For self-adjoint matrices  $A, B \in M_n$  the notation  $A \leq B$  means that  $B - A \in M_n^+$ . For a real-valued function  $f$  of a real variable and a self-adjoint matrix  $A \in M_n$ , the value  $f(A)$  is understood by means of the functional calculus.

For  $0 \leq t \leq 1$  the  $t$ -weighted geometric mean of  $A$  and  $B$  is defined as

$$A\sharp_t B = A^{1/2}(A^{-1/2}BA^{-1/2})^t A^{1/2}.$$

The geometric mean  $A\sharp B := A\sharp_{1/2} B$  is the midpoint of the unique geodesic  $A\sharp_t B$  connecting two points  $A$  and  $B$  in the Riemannian manifold of positive definite matrices with trace metric  $ds = \|A^{-1/2}dA A^{-1/2}\|_F = (\text{Tr}(A^{-1}dA)^2)^{1/2}$  (cf. [7] or [8]).

Recently, Bhatia et al. [1] proved that for any positive definite matrices  $A$  and  $B$  and for  $p = 1, 2$ ,

$$\|A + B + 2rA\sharp B\|_p \leq \|A + B + r(A^{1/2}B^{1/2} + B^{1/2}A^{1/2})\|_p. \quad (1)$$

When  $r = 1$ , inequality holds for  $p = \infty$ .

For the case  $p = 2$  the proof of (1) is based on the following fact: for any positive definite matrices  $A$  and  $B$ ,

$$\lambda(A^{1/2}(A\sharp B)A^{1/2}) \prec_{\log} \lambda(A^{3/4}B^{1/2}A^{3/4}), \quad (2)$$

where the notation  $\lambda$  is used for the  $n$ -tuple of eigenvalues of a matrix  $A$  in decent order and  $\lambda(A) \prec_{\log} \lambda(B)$  means that

$$\prod_{j=1}^k \lambda_j(A) \leq \prod_{j=1}^k \lambda_j(B), \quad 1 \leq k \leq n$$

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