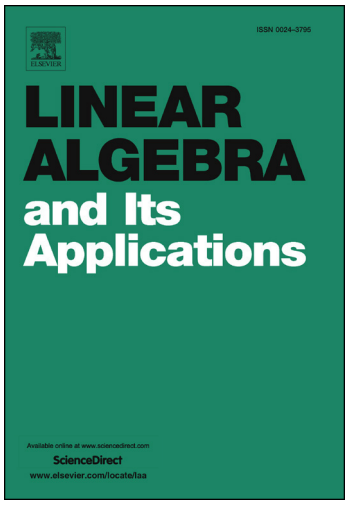

Log-majorization of the moduli of the eigenvalues of a matrix polynomial by tropical roots

Marianne Akian, Stéphane Gaubert, Meisam Sharify

PII: S0024-3795(16)30520-1
DOI: http://dx.doi.org/10.1016/j.laa.2016.11.004
Reference: LAA 13921

To appear in: *Linear Algebra and its Applications*

Received date: 3 May 2016
Accepted date: 4 November 2016

Please cite this article in press as: M. Akian et al., Log-majorization of the moduli of the eigenvalues of a matrix polynomial by tropical roots, *Linear Algebra Appl.* (2017), http://dx.doi.org/10.1016/j.laa.2016.11.004

Log-majorization of the moduli of the eigenvalues of a matrix polynomial by tropical roots

Marianne Akian[a,1], Stéphane Gaubert[a,1,2], Meisam Sharify[b,3,2]

[a] *INRIA and CMAP, École polytechnique, UMR 7641, CNRS. Address: CMAP, École polytechnique, Route de Saclay, 91128 Palaiseau Cedex, France.*
[b] *School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box:19395-5746, Tehran, Iran.*

Abstract

We show that the sequence of moduli of the eigenvalues of a matrix polynomial is log-majorized, up to universal constants, by a sequence of "tropical roots" depending only on the norms of the matrix coefficients. These tropical roots are the non-differentiability points of an auxiliary tropical polynomial, or equivalently, the opposites of the slopes of its Newton polygon. This extends to the case of matrix polynomials some bounds obtained by Hadamard, Ostrowski and Pólya for the roots of scalar polynomials. We also obtain new bounds in the scalar case, which are accurate for "fewnomials" or when the tropical roots are well separated.

Keywords: Matrix polynomial, Tropical algebra, Majorization of eigenvalues, Tropical roots, Roots of polynomial, Bound of Pólya.
2000 MSC: 15A22,15A80,15A18,47J10

1. Introduction

Let $p(x) = \sum_{j=0}^{n} a_j x^j, \quad a_j \in \mathbb{C}$ be a polynomial of degree n in a complex variable x. Let $\zeta_1, \ldots, \zeta_n$ denote the roots of $p(x)$ arranged by non-decreasing modulus (i.e., $|\zeta_1| \leqslant \ldots \leqslant |\zeta_n|$). We associate with p the *tropical polynomial* $\mathtt{t}\,p(x)$, defined for all nonnegative numbers x by

$$\mathtt{t}\,p(x) := \max_{0 \leqslant j \leqslant n} |a_j| x^j \ .$$

Email addresses: Marianne.Akian@inria.fr (Marianne Akian), Stephane.Gaubert@inria.fr (Stéphane Gaubert), meisam.sharify@ipm.ir (Meisam Sharify)

[1] The first and second authors were partially supported by PGMO, a joint program of EDF and FMJH (Fondation Mathématique Jacques Hadamard).

[2] The last two authors were partly supported by the Arpege programme of the French National Agency of Research (ANR), project "ASOPT", number ANR-08-SEGI-005.

[3] This work was performed when the third author was with INRIA Saclay–Île-de-France and CMAP, Ecole polytechnique.