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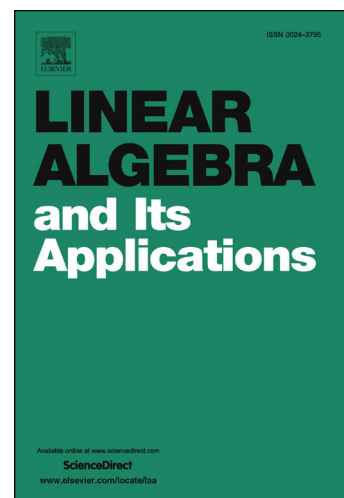
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Log-majorization of the moduli of the eigenvalues of a matrix polynomial by tropical roots

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Abstract

We show that the sequence of moduli of the eigenvalues of a matrix polynomial is log-majorized, up to universal constants, by a sequence of “tropical roots” depending only on the norms of the matrix coefficients. These tropical roots are the non-differentiability points of an auxiliary tropical polynomial, or equivalently, the opposites of the slopes of its Newton polygon. This extends to the case of matrix polynomials some bounds obtained by Hadamard, Ostrowski and Pólya for the roots of scalar polynomials. We also obtain new bounds in the scalar case, which are accurate for “fewnomials” or when the tropical roots are well separated.

Keywords: Matrix polynomial, Tropical algebra, Majorization of eigenvalues, Tropical roots, Roots of polynomial, Bound of Pólya.

2000 MSC: 15A22,15A80,15A18,47J10

1. Introduction

Let $p(x) = \sum_{j=0}^n a_j x^j$, $a_j \in \mathbb{C}$ be a polynomial of degree n in a complex variable x . Let $\zeta_1, \dots, \zeta_n$ denote the roots of $p(x)$ arranged by non-decreasing modulus (i.e., $|\zeta_1| \leq \dots \leq |\zeta_n|$). We associate with p the *tropical polynomial* $\mathbf{t}p(x)$, defined for all nonnegative numbers x by

$$\mathbf{t}p(x) := \max_{0 \leq j \leq n} |a_j| x^j.$$

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