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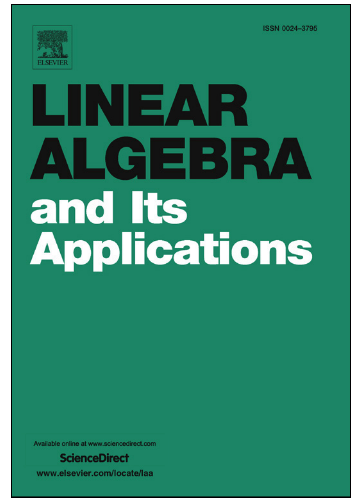

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Spectra of general hypergraphs

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Abstract

Here, we show a method to reconstruct connectivity hypermatrices of a general hypergraph (without any self loop or multiple edge) using tensor. We also study the different spectral properties of these hypermatrices and find that these properties are similar for graphs and uniform hypergraphs. The representation of a connectivity hypermatrix that is proposed here can be very useful for the further development in spectral hypergraph theory.

AMS classification: 05C65, 15A18
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1 Introduction

Spectral graph theory has a long history behind its development. In spectral graph theory, we analyse the eigenvalues of a connectivity matrix which is uniquely defined on a graph. Many researchers have had a great interest to study the eigenvalues of different connectivity matrices, such as, adjacency matrix, Laplacian matrix, signless Laplacian matrix, normalized Laplacian matrix, etc. Now, a recent trend has been developed to explore spectral hypergraph theory. Unlike in a graph, an edge of a hypergraph can be constructed with more than two vertices, i.e., the edge set of a hypergraph is the subset of the power set of the vertex set of that hypergraph [22]. Now, one of the main challenges is to uniquely represent a hypergraph by a connectivity hypermatrix or by a tensor, and vice versa. It is not trivial for a non-uniform hypergraph, where the cardinalities of the edges are not the same. Recently, the study of the spectrum of uniform hypergraph becomes popular. In a (m-) uniform hypergraph, each edge contains the same, (m), number of vertices. Thus an m-uniform hypergraph of order n can be easily represented by an m order n dimensional connectivity hypermatrix (or tensor). In [7], the results on the spectrum of adjacency matrix of a graph are extended for uniform hypergraphs by using characteristic polynomial. Spectral properties of adjacency uniform hypermatrix are deduced from matroids in [16]. In 1993, Fan Chung defined Laplacian of a uniform hypergraph by considering various homological aspects of hypergraphs and studied the eigenvalues of the same [5]. In [9, 10, 11, 18, 19], different spectral properties of Laplacian and signless Laplacian of a uniform hypergraph, defined by using tensor, have been studied. In 2015, Hu and Qi introduced the normalized Laplacian of a uniform hypergraph and analyzed its spectral properties [8]. The important tool that has been used in spectral hypergraph theory is tensor. In 2005, Liqun Qi introduced the different eigenvalues of a real supersymmetric tensor [17]. The various properties of the eigenvalues of a tensor have been studied in [3, 4, 13, 14, 20, 21, 23, 24].

But, still the challenge remains to come up with a mathematical framework to construct a connectivity hypermatrix for a non-uniform hypergraph, such that, based on this connectivity hypermatrix the spectral graph theory