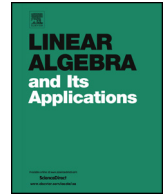




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Weighted least squares solutions of the equation $AXB - C = 0$



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ABSTRACT

Let $\mathcal{H}$ be a Hilbert space, $L(\mathcal{H})$ the algebra of bounded linear operators on $\mathcal{H}$ and $W \in L(\mathcal{H})$ a positive operator such that $W^{1/2}$ is in the p -Schatten class, for some $1 \leq p < \infty$. Given $A, B \in L(\mathcal{H})$ with closed range and $C \in L(\mathcal{H})$, we study the following weighted approximation problem: analyze the existence of

$$\min_{X \in L(\mathcal{H})} \|AXB - C\|_{p,W}, \quad (0.1)$$

where $\|X\|_{p,W} = \|W^{1/2}X\|_p$. We also study the related operator approximation problem: analyze the existence of

$$\min_{X \in L(\mathcal{H})} (AXB - C)^*W(AXB - C), \quad (0.2)$$

where the order is the one induced in $L(\mathcal{H})$ by the cone of positive operators. In this paper we prove that the existence of the minimum of (0.2) is equivalent to the existence of a solution of the normal equation $A^*W(AXB - C) = 0$. We also give sufficient conditions for the existence of the minimum

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of (0.1) and we characterize the operators where the minimum is attained.

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1. Introduction

In signal processing language, *sampling* is an operation which converts a continuous signal (modelled as a vector in an adequate Hilbert space $\mathcal{H}$) into a discrete one.

Frequently the samples of a signal $f \in \mathcal{H}$ are represented in the following way: given a frame $\{v_n\}_{n \in \mathbb{N}} \subseteq \mathcal{H}$ of a closed subspace $\mathcal{S}$, called the sampling subspace, the samples are given by $\{f_n\}_{n \in \mathbb{N}} = \{\langle f, v_n \rangle\}_{n \in \mathbb{N}} \in \ell^2(\mathbb{N})$. On the other hand, given samples $\{f_n\}_{n \in \mathbb{N}} \in \ell^2(\mathbb{N})$, the reconstructed signal $\hat{f}$ is given by $\hat{f} = \sum_{n \in \mathbb{N}} f_n w_n$, where $\{w_n\}_{n \in \mathbb{N}}$ is a frame of the closed subspace $\mathcal{R}$, called the reconstruction subspace.

Suppose that, A and B are the *synthesis operators* corresponding to the frames $\{w_n\}_{n \in \mathbb{N}}$ and $\{v_n\}_{n \in \mathbb{N}}$, respectively, i.e. $A, B : \ell^2(\mathbb{N}) \rightarrow \mathcal{H}$ are the operators such that, if $x = \{x_n\}_{n \in \mathbb{N}} \in \ell^2(\mathbb{N})$, $Ax = \sum_{n \in \mathbb{N}} x_n w_n$ and $Bx = \sum_{n \in \mathbb{N}} x_n v_n$, which are bounded since $\{v_n\}_{n \in \mathbb{N}}$ and $\{w_n\}_{n \in \mathbb{N}}$ are frames. Observe that, the samples of f are given by $\{f_n\}_{n \in \mathbb{N}} = B^* f$ and, given samples $\{f_n\}_{n \in \mathbb{N}}$ the reconstructed signal is given by $\hat{f} = A(\{f_n\}_{n \in \mathbb{N}})$, see [13,25].

If we only know the samples of a signal $\{f_n\}_{n \in \mathbb{N}} \in \ell^2(\mathbb{N})$, in general it is not possible to recover the signal $f \in \mathcal{H}$, even if we apply a digital filter (a bounded linear operator $X : \ell^2(\mathbb{N}) \rightarrow \ell^2(\mathbb{N})$) to these samples. But, in some cases it is possible to find a good representation of the signal $f \in \mathcal{H}$, i.e., a recovered signal $\hat{f} = AXB^* f$ that has good properties. For instance, in the classical sampling scheme (where sampling and reconstruction subspaces coincide) it is possible to reconstruct the best approximation of the signal f , i.e., it is possible to find X such that $AXB^* = P_{\mathcal{S}}$ and then $\hat{f} = P_{\mathcal{S}} f$, where $P_{\mathcal{S}}$ is the orthogonal projection onto $\mathcal{S} = \mathcal{R} = R(A)$. Another interesting example, where the sampling and reconstruction subspaces may not coincide, is the so called consistent sampling scheme, where the samples of the reconstructed signal $\hat{f}$ are equal to the samples of the original signal f , i.e. $B^* \hat{f} = B^* f$, in this case X is such that $Q = AXB^*$ turns out to be an oblique projection. Consequently, the reconstructed signal $\hat{f}$ is not necessarily a good approximation of f , since the distance $\|f - \hat{f}\| = \|f - AXB^* f\|$ is not minimized. Now suppose we want to find a digital filter $X \in \ell^2(\mathbb{N}) \rightarrow \ell^2(\mathbb{N})$ such that $AXB^* f$ is a good approximation of f in $R(A) = \mathcal{R}$, i.e. we want that AXB^* approximates $P_{\mathcal{R}}$ in some sense. For instance, we may want to find $X_0 : \ell^2(\mathbb{N}) \rightarrow \ell^2(\mathbb{N})$ a bounded linear operator, such that, for every $f \in \mathcal{H}$

$$\|(AX_0 B^* - P_{\mathcal{R}})f\| \leq \|(AXB^* - P_{\mathcal{R}})f\|,$$

for every $X \in L(\mathcal{H})$ (the algebra of linear bounded operators on $\mathcal{H}$). This means that we are interested in the problem:

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