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## Existence of a not necessarily symmetric matrix with given distinct eigenvalues and graph



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### ARTICLE INFO

#### Article history:

Received 7 April 2016

Accepted 3 April 2017

Available online 4 April 2017

Submitted by J.F. Queiro

#### MSC:

05C50

15A18

15A29

15B57

65F18

#### Keywords:

Inverse eigenvalue problem

Graph

Jacobian method

Implicit function theorem

Transversal intersection

### ABSTRACT

For given distinct numbers  $\lambda_1 \pm \mu_1 i, \lambda_2 \pm \mu_2 i, \dots, \lambda_k \pm \mu_k i \in \mathbb{C} \setminus \mathbb{R}$  and  $\gamma_1, \gamma_2, \dots, \gamma_l \in \mathbb{R}$ , and a given graph  $G$  with a matching of size at least  $k$ , we will show that there is a real matrix whose eigenvalues are the given numbers and its graph is  $G$ . In particular, this implies that any real matrix with distinct eigenvalues is similar to a real, irreducible, tridiagonal matrix.

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<sup>1</sup> The work of the author was partially supported by the Natural Sciences and Engineering Research Council of Canada.

### 1. Introduction

A *directed graph*  $G = (V, E)$  is a pair of sets  $V$  and  $E$  where  $V$  is the set of vertices of  $G$ , and  $E$ , the set of edges of  $G$ , is a subset of  $V \times V$ . That is, each element of  $E$  is an ordered pair  $(u, v)$ , with  $u, v \in V$ . We say a graph  $G$  is *loopless* when for each  $(u, v) \in E$ , we have  $u \neq v$ . In this paper we only consider loopless graphs. If  $(u, v) \in E$  then we say  $u$  is adjacent to  $v$  and denote it by  $u \rightarrow v$ . Note that such a graph might have both edges  $(u, v)$  and  $(v, u)$ , but since  $E$  is a set, there are no multiple edges from  $u$  to  $v$ . A directed loopless graph  $G = (V, E)$  is said to have a *matching* of size  $k$  if  $E$  contains  $k$  vertex-disjoint edges  $(u_1, v_1), \dots, (u_k, v_k)$  and their reverses  $(v_1, u_1), \dots, (v_k, u_k)$ .

A graph  $G$  is called *undirected* if for each  $u \neq v$  the edge  $(u, v) \in E$  if and only if  $(v, u) \in E$ . Hence we can ignore the directions of edges and consider  $E$  as a set of 2-subsets of  $V$ . That is,  $E \subset \{\{u, v\} \mid u, v \in V\}$ . In this case we call  $G$  an *undirected graph*. An undirected loopless graph  $G = (V, E)$  is said to have a *matching* of size  $k$  if  $E$  contains  $k$  vertex-disjoint edges  $\{u_1, v_1\}, \dots, \{u_k, v_k\}$

Let  $A \in \mathbb{R}^{n \times n}$ . We say a (directed or undirected) loopless graph  $G$  is the *graph of the matrix*  $A$  when for each  $i \neq j$  we have  $A_{i,j} \neq 0$  if and only if  $i \rightarrow j$ . Note that the diagonal entries of  $A$  can be zero or nonzero.

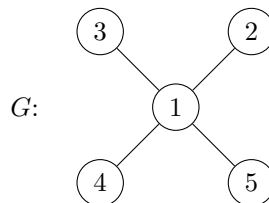
It is of interest to study the existence of matrices with given spectral properties and graph, see [1, Chapter 4]. For the problems when the solution matrix is not necessarily symmetric see [2] for a survey on the structured inverse eigenvalue problems with an extensive bibliography, specially SIEP6b, and see [3] for the related minimum rank problems. In this paper we examine a the problem of existence of a solution matrix  $A \in \mathbb{R}^{n \times n}$  where  $G$ , the graph of  $A$ , and  $\Lambda$ , its spectrum, are prescribed. An obvious necessary condition for the existence of a solution is that  $\Lambda$  to be closed under complex conjugation. Assume that  $\Lambda$  consists of  $k$  distinct complex conjugate pairs in  $\mathbb{C} \setminus \mathbb{R}$ . We will show that a sufficient condition for the existence of a solution is that

- C1.**  $G$  has a matching of size at least  $k$ , and
- C2.** all the eigenvalues are distinct.

Example 1.1 shows that condition C1 is not necessary, and Example 1.2 shows that condition C2 is not necessary.

**Example 1.1.** Let  $G$  be a star on 5 vertices with 1 as the center vertex, and let

$$A = \begin{bmatrix} 3 & -2 & -6 & 1 & 6 \\ 5 & -2 & 0 & 0 & 0 \\ -5 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 3 & 0 \\ -6 & 0 & 0 & 0 & 5 \end{bmatrix},$$



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