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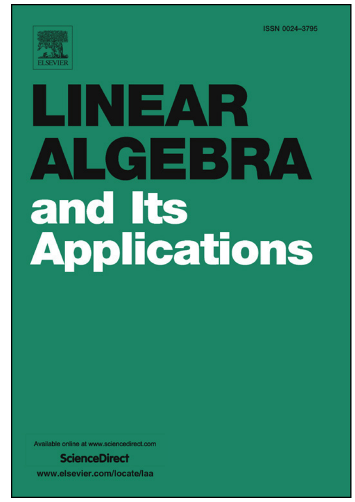

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Abstract

For each idempotent E in the compact affine semigroup $\mathbf{St}_n$ of $n \times n$ stochastic matrices the *core* of E, denoted by $\mathbf{Q}(E)$, is defined as the maximal subsemigroup of $\mathbf{St}_n$ with zero element E. In this paper we study the algebraic and geometric structures of cores and describe how they are put together inside of $\mathbf{St}_n$. In the process, lattice structures of the idempotents and their cores inside a larger core are revealed and then employed to initiate a possible classification.

Key words:
Affine semigroups
Stochastic matrices
Doubly stochastic matrices
Convex polytopes of matrices

MSC: 22A15, 15B51, 52B12

1. Preliminaries

This work continues the research initiated in [17] on maximal monoids and cores of the semigroups of nonnegative and stochastic matrices. We have taken from that paper notation, definitions and results to apply them throughout. The approach we have been taking places this work in the context of affine semigroups, that is, convex sets $\mathbf{S}$ on a real topological vector space with an associative binary operation that respects convex combinations of its elements.

Denote by $\mathbf{M}_n$ the algebra of $n \times n$ real matrices. Let $\mathbf{St}_n$ denote the set of $n \times n$ stochastic matrices. This constitutes a compact affine subsemigroup of $\mathbf{M}_n$ with identity I_n. It is also a convex polytope of dimension $n(n-1)$ whose vertices are all the stochastic matrices (n^n in total) having either 0 or 1 in any of their entries. Similarly, the set $\mathbf{D}_n$ of $n \times n$ doubly stochastic matrices constitutes a compact affine subsemigroup of $\mathbf{St}_n$ with identity I_n, and a convex polytope of dimension $(n-1)^2$ whose vertices are the permutation matrices, which are known to form a group $\mathbf{P}_n$ isomorphic to the symmetric group $\mathcal{S}_n$ on n letters, and hence there are $n!$ of them.

Since $\mathbf{St}_n$ is a compact semigroup it has a unique minimal ideal $\mathbf{M}(\mathbf{St}_n)$. This is a closed, hence compact, right-trivial subsemigroup of $\mathbf{St}_n$, whose elements are the stochastic matrices

[0] To Esthela, my dear mother, with deep and constant gratitude.