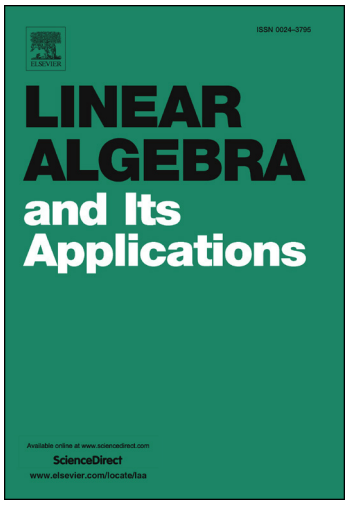


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# Generalization of Roth's solvability criteria to systems of matrix equations

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**Abstract**

W.E. Roth (1952) proved that the matrix equation $AX - XB = C$ has a solution if and only if the matrices $\left[\begin{smallmatrix} A & C \\ 0 & B \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} A & 0 \\ 0 & B \end{smallmatrix}\right]$ are similar. A. Dmytryshyn and B. Kågström (2015) extended Roth's criterion to systems of matrix equations $A_i X_{i'} M_i - N_i X_{i''}^{\sigma_i} B_i = C_i$ $(i = 1, \dots, s)$ with unknown matrices $X_1, \dots, X_t$, in which every $X^\sigma$ is $X$, $X^\top$, or $X^*$. We extend their criterion to systems of complex matrix equations that include the complex conjugation of unknown matrices. We also prove an analogous criterion for systems of quaternion matrix equations.

*AMS classification:* 15A24

*Keywords:* Systems of matrix equations, Sylvester equations, Roth's criteria

## 1. Introduction

Roth [15] proved that the matrix equation $AX - XB = C$ (respectively, $AX - YB = C$) over a field has a solution if and only if the matrices $\left[\begin{smallmatrix} A & C \\ 0 & B \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} A & 0 \\ 0 & B \end{smallmatrix}\right]$ are similar (respectively, equivalent); see also [8, Section 4.4.22] and [11, Section 12.5].

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