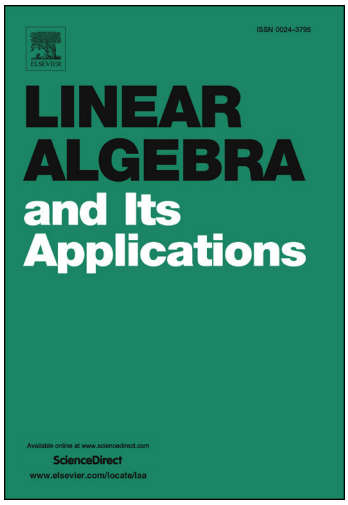

Imprimitivity Index of the Adjacency Matrix of Digraphs

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Imprimitivity Index of the Adjacency Matrix of Digraphs

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Abstract

Let G be a graph. An edge orientation of G is called smooth if the in-degree and the out-degree of every vertex differ by at most one. In this paper, we show that if G is a 2-edge-connected non-bipartite graph with $\delta(G) \geq 3$, then G has a smooth primitive orientation. Among other results, using the spectral radius of digraphs, we show that if D_1 is a primitive regular orientation and D_2 is a non-regular orientation of a given graph, then for sufficiently large t, the number of closed walks of length t in D_1 is more than the number of closed walks of length t in D_2.

Keywords: Primitive digraph, Imprimitivity index, Number of closed walks, Spectral radius.

2010 Mathematics Subject Classification: 05C20, 05C50, 15A18.

1 Introduction

In this article we follow the definitions and terminologies of [1, 2, 3]. Throughout this paper, all graphs are simple with no loops or multiple edges. Let G be a graph. The set of vertices and the set of edges of G are denoted by $V(G)$ and $E(G)$, respectively. The minimum degree of the vertices of G is denoted by $\delta(G)$. An *even graph* is a graph in which the degree of each vertex is even. The complete graph and the cycle with n vertices are denoted by K_n and C_n, respectively.

A *directed graph* or briefly a *digraph* D consists of a non-empty finite set $V(D)$ of elements called *vertices* and a finite set $E(D)$ of ordered pairs of distinct vertices called *arcs* or *edges*. An *orientation* of a graph G is a digraph whose underlying graph is G. A digraph D is *regular* of

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