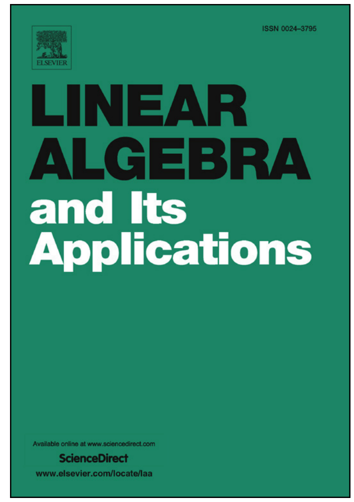

Eigenvalues of Periodic Sturm Liouville Problems

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Abstract. For h-periodic coefficients and any integer k>2 it is well known that the eigenvalues of some self-adjoint complex boundary condition on the interval [a, a+h] are the same as the periodic eigenvalues on the interval [a, a+kh]. For each k we identify explicitly which of the uncountable number of complex conditions generates these periodic eigenvalues. In addition, we prove an analogous result for semi-periodic eigenvalues.

1. Introduction

We study eigenvalues of the equation

$$-(py')' + qy = \lambda wy,\ \lambda \in \mathbb{C}, \tag{1.1}$$

with coefficients satisfying

$$1/p,\ q,\ w \in L_{loc}(\mathbb{R},\mathbb{R}),\ p > 0,\ w > 0\ a.e.\ on\ \mathbb{R}, \tag{1.2}$$

and for some $h \in \mathbb{R}$, $0 < h < \infty$ the coefficients are $h-$periodic:

$$p(t+h) = p(t),\ q(t+h) = q(t),\ w(t+h) = w(t),\ t \in \mathbb{R}; \tag{1.3}$$

together with the complex boundary conditions

$$\begin{aligned} y(a+h) &= e^{i\gamma}y(a) \\ (py')(a+h) &= e^{i\gamma}(py')(a),\ 0 < \gamma < \pi \end{aligned} \tag{1.4}$$

and, for any $a \in \mathbb{R}$, the periodic and semi-periodic boundary conditions on the k-intervals $[a + kh]$, $k \in \mathbb{N}_1 = \{1, 2, 3, \cdots\}$:

$$\begin{aligned} y(a+kh) &= y(a) \\ (py')(a+kh) &= (py')(a), \end{aligned} \tag{1.5}$$

$$\begin{aligned} y(a+kh) &= -y(a) \\ (py')(a+kh) &= -(py')(a). \end{aligned} \tag{1.6}$$

Note that condition (1.2) implies that the coefficients satisfy

$$1/p,\ q,\ w \in L^1(J,\mathbb{R}),\ p > 0,\ w > 0\ a.e.\ on\ J, \tag{1.7}$$

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