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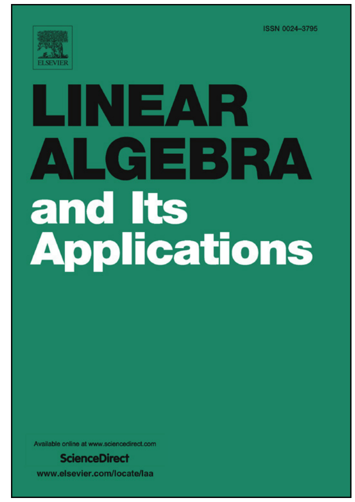

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Bounds of graph energy in terms of vertex cover number *

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Abstract: The energy $E(G)$ of a graph G is the sum of the absolute values of all eigenvalues of G. In this paper, we give a lower bound and an upper bound for graph energy in terms of vertex cover number. For a graph G with vertex cover number τ, it is proved that $2\tau - 2c \leq E(G) \leq 2\tau\sqrt{\Delta}$, where c is the number of odd cycles in G and Δ is the maximum vertex degree of G. The lower bound is attained if and only if G is the disjoint union of some complete bipartite graphs with perfect matchings and some isolated vertices, the upper bound is attained if and only if G is the disjoint union of τ copies of $K_{1,\Delta}$ together with some isolated vertices.

AMS classification: 05C20, 05C50, 05C75.

Keywords: Graph energy; Vertex cover number; Matching number

1 Introduction

Let G be an undirected graph without multiple edges and loops. The energy $E(G)$ of G is defined to be the sum of the absolute values of all eigenvalues of $A(G)$, where $A(G)$ denotes the adjacency matrix of G. The motivation for the definition of $E(G)$ comes from chemistry, where the first results on $E(G)$ were obtained as early as the 1940s [1]. However, in the last two decades research on graph energy has much intensified, resulting in a large number of publications. For detailed results on graph energy we refer the reader to book [2], where the authors summarized the most important results involving graph energy. We here only introduce the publications studying the bounds of graph energy. Caporossi et al. [3] proved that $E(G) \geq 2\sqrt{m}$ for all graphs G with m edges. Rada [4] extended the above result to a diagraph D by proving that if a digraph D has c_2 closed walks of length 2 then the energy of D is not less than $\sqrt{2c_2}$. Rada and Tineo [5] proved that $E(G) \geq 2m\sqrt{\frac{m}{q}}$ for a bipartite graph G with m edges and with q

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