Linear Algebra and its Applications 517 (2017) 235–245

Contents lists available at ScienceDirect

Linear Algebra and its Applications

www.elsevier.com/locate/laa

Joining models with commutative orthogonal block structure

Carla Santos[a,b,*], Célia Nunes[c], Cristina Dias[d,b], João Tiago Mexia[e,b]

[a] *Department of Mathematics and Physical Sciences, Polytechnical Institute of Beja, Rua Pedro Soares, Campus do IPBeja, Beja, Portugal*
[b] *CMA – Center of Mathematics and its Applications, FCT/UNL, Portugal*
[c] *Department of Mathematics and Center of Mathematics and Applications, University of Beira Interior, Portugal*
[d] *Polytechnical Institute of Portalegre, School of Technology and Management, Portugal*
[e] *Department of Mathematics, Faculty of Science and Technology, New University of Lisbon, Portugal*

ARTICLE INFO

Article history:
Received 9 January 2016
Accepted 14 December 2016
Available online 16 December 2016
Submitted by W.B. Wu

MSC:
62J10
17C65

Keywords:
Jordan algebra
Mixed models
Models with commutative orthogonal block structure
Models joining

ABSTRACT

Mixed linear models are a versatile and powerful tool for analysing data collected in experiments in several areas. A mixed model is a model with orthogonal block structure, OBS, when its variance–covariance matrix is of all the positive semi-definite linear combinations of known pairwise orthogonal orthogonal projection matrices that add up to the identity matrix. Models with commutative orthogonal block structure, COBS, are a special case of OBS in which the orthogonal projection matrix on the space spanned by the mean vector commutes with the variance–covariance matrix.

Using the algebraic structure of COBS, based on Commutative Jordan algebras of symmetric matrices, and the Cartesian product we build up complex models from simpler ones through joining, in order to analyse together models obtained independently. This commutativity condition of COBS is a necessary and sufficient condition for the least square estimators, LSE, to be best linear unbiased estimators, BLUE, whatever the variance components. Since joining COBS we

* Corresponding author at: Department of Mathematics and Physical Sciences, Polytechnical Institute of Beja, Rua Pedro Soares, Campus do IPBeja, Beja, Portugal.
E-mail address: carla.santos@ipbeja.pt (C. Santos).

http://dx.doi.org/10.1016/j.laa.2016.12.019

obtain new COBS, the good properties of estimators hold for the joined models.

1. Introduction

Mixed models are a versatile and powerful tool for analysing data collected in experiments, see for example [12], and, over the years, they have been applied to several areas such as biological and medical research, animal and human genetics, agriculture or industry.

A mixed model whose variance–covariance matrix is of all the positive semi-definite linear combinations,

$$\sum_{j=1}^{m} \gamma_j \boldsymbol{Q}_j$$

of known pairwise orthogonal orthogonal projection matrices, POOPM, $\boldsymbol{Q}_1 \dots \boldsymbol{Q}_m$, that up to $\boldsymbol{I}_n$, is a model with orthogonal block structure, OBS. These models, introduced by Nelder [16,17], have been intensively studied [10,13] and continue to play a central part in the theory of randomised block designs [2,3].

OBS in which the matrices $\boldsymbol{Q}_1 \dots \boldsymbol{Q}_m$ commute with $\boldsymbol{T}$, the orthogonal projection matrix on the space spanned by the mean vector, are called models with commutative orthogonal block structure, COBS. This special class of OBS, was introduced in [9] and has also been considered by Santos et al. [19], Nunes et al. [18], Carvalho et al. [4], Ferreira et al. [7], Carvalho et al. [5] and Bailey et al. [1]. Therefore COBS are models with OBS whose variance–covariance matrix commutes with the orthogonal projection matrix on the space spanned by the mean vector. This commutativity condition is a necessary and sufficient condition for the least square estimators, LSE, to be best linear unbiased estimators, BLUE, whatever the variance components [23].

In order to build up complex models from simple ones, Mexia et al. [15] introduced models crossing and models nesting, two operations between models based on the binary operations on CJAS, Kronecker product of CJAS and the restricted Kronecker product of CJAS, introduced in Fonseca et al. [8].

Now we introduce models joining, a possible alternative to models crossing and models nesting, with the same purpose to analyse together models obtained independently.

Let $\boldsymbol{y}(1) \dots \boldsymbol{y}(n)$ be the observations vectors of n models with null cross-covariance matrices, then

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}(1) \\ \vdots \\ \boldsymbol{y}(n) \end{bmatrix}$$