

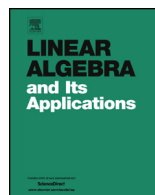


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## On the boundaries of strict pseudospectra <sup>☆</sup>



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### ABSTRACT

The boundary of the ordinary  $\varepsilon$ -pseudospectrum of a square matrix is contained in the boundary of the strict  $\varepsilon$ -pseudospectrum. This content relation may be strict in some cases.

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## 1. Introduction

The boundary of the ordinary pseudospectrum of a square matrix  $A$  of level  $\varepsilon$ , denoted by  $\partial\Lambda_\varepsilon(A)$ , is contained in the boundary of the strict pseudospectrum of the same level

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$\partial\Lambda'_\varepsilon(A)$  (see Remark 3.2, p. 280 in [1]). In this paper we will prove that in general these boundaries are not equal.

An equivalent problem is to determine whether the function of  $z \mapsto \sigma_n(zI_n - A)$  can have local maxima. Thus, we will show that a complex number  $z_0 \in \partial\Lambda'_\varepsilon(A) \setminus \partial\Lambda_\varepsilon(A)$  if and only if the function  $z \mapsto \sigma_n(zI_n - A)$  reaches a local maximum at  $z_0$ . As a result, we will prove that the function  $z \mapsto \sigma_n(zI_n - A)$  can have local maxima.

On the other hand, both the ordinary pseudospectrum of a matrix  $A$  of level  $\varepsilon$  and its boundary are semialgebraic sets [6]. We will prove that this property is also true for the strict pseudospectrum. This fact will allow us to prove that the set  $\partial\Lambda'_\varepsilon(A) \setminus \partial\Lambda_\varepsilon(A)$  can be: empty, finite, or formed by the union of a finite set and a real analytic submanifold of dimension 1 with a finite number of connected components.

## 2. Previous notation and main results

For the inclusion relation between two sets  $X$  and  $Y$  we will use the notations  $X \subset Y$  and  $X \subsetneq Y$  to mean “ $X$  is contained in or equal to  $Y$ ” and “ $X$  is strictly contained in  $Y$ ”, respectively. Let  $\mathbb{C}^{n \times n}$  denote the space of  $n \times n$  complex matrices. For any matrix  $M \in \mathbb{C}^{n \times n}$  let

$$\sigma_1(M) \geq \sigma_2(M) \geq \dots \geq \sigma_n(M)$$

denote its singular values in decreasing order. Let  $\Lambda(A)$  denote the spectrum of the matrix  $A \in \mathbb{C}^{n \times n}$ . Given  $A \in \mathbb{C}^{n \times n}$  and  $\varepsilon > 0$  the *ordinary pseudospectrum of level  $\varepsilon$*  is the set

$$\Lambda_\varepsilon(A) := \bigcup_{\|\Delta\| \leq \varepsilon} \Lambda(A + \Delta),$$

where  $\|\cdot\|$  denotes the spectral norm. Analogously, the *strict pseudospectrum of level  $\varepsilon$*  is the set

$$\Lambda'_\varepsilon(A) := \bigcup_{\|\Delta\| < \varepsilon} \Lambda(A + \Delta).$$

Let  $g$  denote the function

$$g(z) := \sigma_n(zI_n - A), \quad z \in \mathbb{C}; \tag{1}$$

using this function a characterization of the pseudospectra is given by

$$\Lambda_\varepsilon(A) = \{z \in \mathbb{C} : g(z) \leq \varepsilon\}, \tag{2}$$

and

$$\Lambda'_\varepsilon(A) = \{z \in \mathbb{C} : g(z) < \varepsilon\}. \tag{3}$$

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