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# On vector spaces of linearizations for matrix polynomials in orthogonal bases

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ABSTRACT

Regular and singular matrix polynomials $P(\lambda) = \sum_{i=0}^{k} P_i\phi_i(\lambda), P_i \in \mathbb{R}^{n\times n}$ given in an orthogonal basis $\phi_0(\lambda), \phi_1(\lambda), \ldots, \phi_k(\lambda)$ are considered. Following the ideas in [9], the vector spaces, called $\mathbb{M}_1(P)$, $\mathbb{M}_2(P)$ and $\mathbb{DM}(P)$, of potential linearizations for $P(\lambda)$ are analyzed. All pencils in $\mathbb{M}_1(P)$ are characterized concisely. Moreover, several easy to check criteria whether a pencil in $\mathbb{M}_1(P)$ is a (strong) linearization of $P(\lambda)$ are given. The equivalence of some of them to the Z-rank-condition [9] is pointed out. Results on the vector space dimensions, the genericity of linearizations in $\mathbb{M}_1(P)$ and the form of block-symmetric pencils are derived in a new way on a basic algebraic level. Moreover, an extension of these results to degree-graded bases is presented. Throughout the paper, structural resemblances between the matrix pencils in $\mathbb{L}_1$, i.e. the results obtained in [9], and their generalized versions are pointed out.

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## 1. Introduction

Linearization of matrix polynomials expressed in standard and nonstandard bases have received much attention in recent years. In the ground-breaking paper [9] vector spaces of possible linearizations of matrix polynomials have been introduced. These turned out to build an elegant framework to find and construct linearizations for square matrix polynomials as well as to study their algebraical and analytical properties. While the paper [9] is mainly concerned with the characterization and analysis of these spaces for matrix polynomials in the standard monomial basis, recently the research on matrix polynomials and linearizations expressed in nonstandard polynomial bases has received more attention, see, e.g., [1,4–6,8,11–13].

This paper is devoted to the study of regular and singular matrix polynomials $P(\lambda) = \sum_{i=0}^{k} P_i\phi_i(\lambda)$, $P_i \in \mathbb{R}^{n\times n}$ expressed in an orthogonal basis $\{\phi_i(\lambda)\}_{i=0}^{k}$, generalizing most concepts from [9] to this special case. In particular, we will consider the set $\mathbb{M}_1(P)$ of all $kn \times kn$ matrix pencils $\mathcal{L}(\lambda)$ satisfying

$$\mathcal{L}(\lambda)(\Phi_k(\lambda) \otimes I_n) = v \otimes P(\lambda)$$

with $\Phi_k(\lambda) := [\phi_{k-1} \ \cdots \ \phi_1 \ \phi_0]^T$. For the monomial basis, this is just the definition of $\mathbb{L}_1(P)$ [9, Definition 3.1] with $\Phi_k(\lambda) = [\lambda^{k-1} \ \cdots \ \lambda \ 1]^T =: \Lambda_k(\lambda)$. The same kind of generalization of $\mathbb{L}_1(P)$ to matrix polynomials in nonstandard bases has been already considered, e.g., in [4,11]. We will give an explicit characterization of the elements of $\mathbb{M}_1(P)$ that enables us to formulate our results readily accessible providing quite short proofs. Moreover, we show how to easily construct linearizations by means of an intuitive and readily checked linearization condition. Clearly, most of our findings are equivalent to already known results. Thus our main contribution here is a new view aiming to open up new perspectives on the structure of ansatz spaces in general and present even well-known facts in a new livery. A second main goal is to present the facts in a concise and succinct manner keeping the proofs on a basic algebraic level without drawing on deeper theoretical results. We present our results assuming the field underlying our derivations are the real numbers $\mathbb{R}$. However, we expect that most of the concepts immediately extend (appropriately adjusted) to arbitrary fields $\mathbb{F}$, in particular to the complex numbers $\mathbb{C}$.

In Section 2 the basic notation used and some well-known results are summarized. In Section 3, generalized ansatz spaces for orthogonal bases are defined and their basic properties are proven. Section 4 is concerned with the eigenvector recovery, while in Section 5 singular matrix polynomials are considered. The extension of the double ansatz space from [9] to orthogonal bases is the subject of Section 6, whereas Section 7 provides a construction algorithm for block-symmetric pencils. Section 8 presents a partial generalization of the eigenvalue exclusion theorem, while Section 9 is dedicated to the question how the results presented up to Section 8 may be derived when an arbitrary

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