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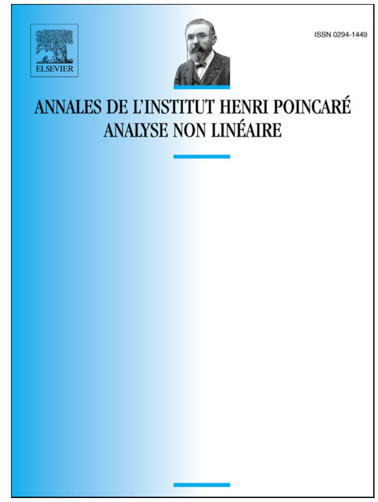

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log-log blow up solutions blow up at exactly m points

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Abstract

We study the focusing mass-critical nonlinear Schrödinger equation, and construct certain solutions which blow up at exactly m points according to the log-log law.

Résumé

Nous étudions l'équation de Schrödinger non linéaire focalisante de masse critique, et construisons certaines solutions avec exactement m points d'explosion d'après la loi de log-log.

Keywords: NLS, log-log blow up, m points blow up, bootstrap, propagation of regularity, topological argument.
2010 MSC: 35Q55, 35B44.

1. Introduction

We consider the Cauchy Problem for the mass-critical focusing nonlinear Schrödinger equation (NLS) on $\mathbb{R}^d$ for $d = 1, 2$:

$$(NLS)\begin{cases} iu_t = -\Delta u - |u|^{\frac{4}{d}}u, \\ u(0) = u_0 \in H^1(\mathbb{R}^d). \end{cases} \tag{1.1}$$

Problem (1.1) has three conservation laws :

— Mass :

$$M(u(t,x)) := \int |u(t,x)|^2 dx = M(u_0), \tag{1.2}$$

— Energy :

$$E(u(t,x)) := \frac{1}{2}\int |\nabla u(t,x)|^2 dx - \frac{1}{2+\frac{4}{d}}\int |u(t,x)|^{2+\frac{4}{d}} dx = E(u_0), \tag{1.3}$$

— Momentum :

$$P(u(t,x)) := \Im(\int \nabla u(t,x)\overline{u(t,x)})dx = P(u_0), \tag{1.4}$$

and the following symmetry :

1. Space-time translation : If $u(t,x)$ solves (1.1), then $\forall t_0 \in \mathbb{R}, x_0 \in \mathbb{R}^d$, we have $u(t-t_0, x-x_0)$ solves (1.1).

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