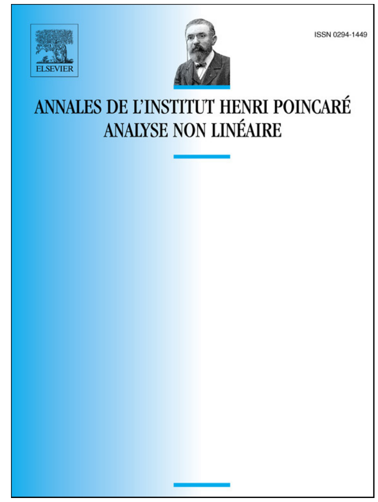

A Keller-Segel type system in Higher Dimensions

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A KELLER-SEGEL TYPE SYSTEM IN HIGHER DIMENSIONS

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Abstract. We analyze an equation that is gradient flow of a functional related to Hardy-Littlewood-Sobolev inequality in whole Euclidean space $\mathbb{R}^d, d \geq 3$. Under the hypothesis of integrable initial data with finite second moment and energy, we show local-in-time existence for any mass of "free-energy solutions", namely weak solutions with some free energy estimates. We exhibit that the qualitative behavior of solutions is decided by a critical value. Actually, there is a critical value of a parameter in the equation below which there is a global-in-time energy solution and above which there exists blowing-up energy solutions.

1. Introduction

There has been recent interest in introducing a higher-dimensional analog of Patlak-Keller-Segel(PKS) system; see [3, 10, 19, 22, 23] and the references therein. The original model is a simplified version of the model that describes the collective motion of cells that are attracted by a self-emitted chemical substance. There are many proposed mathematical models for chemotaxis. As far as we know, the first mathematical model was introduced by Patlak in [21] and later by Keller and Segel in [15]. Further simplification has been proposed later, in which case the equations take the following form which we call the PKS system:

$$\begin{cases} \frac{\partial f}{\partial t}(t,x) = \Delta f(t,x) - \chi \nabla \cdot (f(t,x)\nabla c(t,x)), \quad t>0, x \in \mathbb{R}^2, \\ -\Delta c(t,x) = f(t,x), \quad t>0, x\in \mathbb{R}^2, \\ f(0,x) = f_0(x) \geq 0. \end{cases} \tag{1.1}$$

Here, $(t,x) \mapsto f(t,x)$ is the cell density, and $(t,x) \mapsto c(t,x)$ is the concentration of chemoattractant. The first equation in (1.1) takes into account that the motion of cells is driven by the steepest increase in the concentration of chemoattractant while following a Brownian motion due to external interactions. The second equation in (1.1) takes into account that the cells are producing the chemoattractant themselves and while this is diffusing into the environment.

$\chi > 0$ is the sensitivity of the bacteria to the chemoattractant, assumed to be a constant; which measures the nonlinearity of the system.

The existence of solutions, critical mass phenomena, blow-up of solutions, qualitative behavior of solutions for equation (1.1) and similar equations

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