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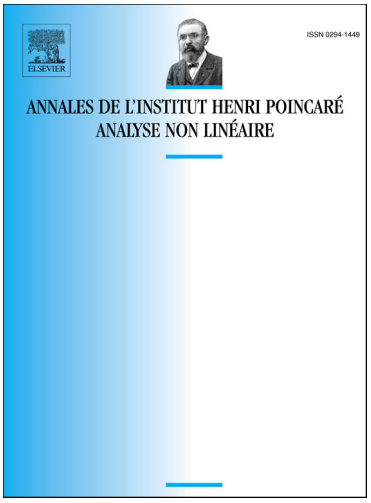

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Global existence for reaction-diffusion systems with nonlinear diffusion and control of mass

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Abstract : We prove here global existence in time of weak solutions for some reaction-diffusion systems with natural structure conditions on the nonlinear reactive terms which provide positivity of the solutions and uniform control of the total mass. The diffusion operators are nonlinear, in particular operators of the porous media type $u_i \mapsto -d_i \Delta u_i^{m_i}$. Global existence is proved under the assumption that the reactive terms are bounded in L^1. This extends previous similar results obtained in the semilinear case when the diffusion operators are linear of type $u_i \mapsto -d_i \Delta u_i$.

Keywords : reaction-diffusion system, nonlinear diffusion, fast diffusion, porous media equation, global existence, weak solution, control of mass

2000 Mathematics Subject Classification : 35K10, 35K40, 35K57

1 Introduction

The goal of this paper is the study of global existence in time of solutions to reaction-diffusion systems of the following type

$$\begin{cases} \text{for all } i = 1, ..., m, & & & \\ \partial_t u_i - \Delta \varphi_i(u_i) & = & f_i(u_1, u_2, \cdots, u_m) & \text{in }]0, +\infty[\times\Omega \\ u_i(t, .) & = & 0, & \text{on }]0, +\infty[\times\partial\Omega, \\ u_i(0, .) & = & u_{i0} \geq 0 & \text{in } \Omega. \end{cases} \tag{1}$$

Here Ω is a bounded open subset of $\mathbb{R}^N$ with a regular boundary, $\varphi_i, i = 1, ..., m$ are continuous increasing functions from $[0, +\infty)$ into $[0, +\infty)$ with $\varphi_i(0) = 0$ and the f_i are regular functions such that the two following main properties occur :

• (P) : the nonnegativity of the solutions is preserved for all time ;

• (M) : the total mass of the components is controlled for all time (sometimes even exactly preserved).

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