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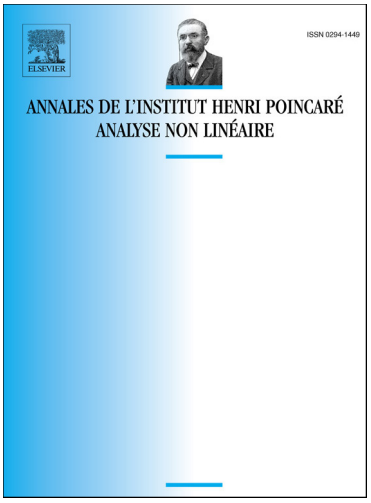

On phase separation in systems of coupled elliptic equations: Asymptotic analysis and geometric aspects

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ON PHASE SEPARATION IN SYSTEMS OF COUPLED ELLIPTIC EQUATIONS: ASYMPTOTIC ANALYSIS AND GEOMETRIC ASPECTS

NICOLA SOAVE AND ALESSANDRO ZILIO

Abstract. We consider a family of positive solutions to the system of k components

$$-\Delta u_{i,\beta} = f(x, u_{i,\beta}) - \beta u_{i,\beta} \sum_{j\neq i} a_{ij} u_{j,\beta}^2 \qquad \text{in } \Omega,$$

where $\Omega \subset \mathbb{R}^N$ with $N \geq 2$. It is known that uniform bounds in L^∞ of $\{\mathbf{u}_\beta\}$ imply convergence of the densities to a segregated configuration, as the competition parameter β diverges to $+\infty$. In this paper we establish sharp quantitative point-wise estimates for the densities around the interface between different components, and we characterize the asymptotic profile of $\mathbf{u}_\beta$ in terms of entire solutions to the limit system

$$\Delta U_i = U_i \sum_{j\neq i} a_{ij} U_j^2.$$

Moreover, we develop a uniform-in-β regularity theory for the interfaces.

1. Introduction

The aim of this paper is to prove qualitative properties of positive solutions to competing systems with variational interaction, whose prototype is the coupled Gross-Pitaevskii equation

$$\begin{cases} -\Delta u_{i,\beta} + \lambda_{i,\beta} u_{i,\beta} = \mu_i u_{i,\beta}^3 - \beta u_{i,\beta} \sum_{j\neq i} a_{ij} u_{j,\beta}^2 & \text{in } \Omega \\ u_i > 0 & \text{in } \Omega, \end{cases} \qquad i = 1, \dots, k,$$

in the limit of strong competition $\beta \to +\infty$. This problem naturally arises in different contexts: from the physics world, it is of interest in nonlinear optics and in the Hartree-Fock approximation for Bose-Einstein condensates with multiple hyperfine states, see e.g. [1, 25]. From a mathematical point of view, it is useful in the approximation of optimal partition problems for Laplacian eigenvalues, and in the theory of harmonic maps into singular manifolds, see [4, 7, 8, 17, 24]. Several papers are devoted to the development of a common regularity theory for families of solutions associated to families of parameters $\beta \to +\infty$, to the analysis of the convergence of such families to some limit profile, and to the regularity issues for the emerging free-boundary problem, see [4, 5, 6, 7, 8, 16, 17, 18, 21, 28]. On the other hand, not much is known about finer qualitative properties, such as:

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