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Classification of Edges using Compactly Supported Shearlets

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Abstract

We analyze the detection and classification of singularities of functions $f = \chi_S$, where $S \subset \mathbb{R}^d$ and $d = 2, 3$. It will be shown how the set ∂S can be extracted by a continuous shearlet transform associated with compactly supported shearlets. Furthermore, if ∂S is a $d-1$ dimensional piecewise smooth manifold with $d = 2$ or 3, we will classify smooth and non-smooth components of ∂S. This improves previous results given for shearlet systems with a certain band-limited generator, since the estimates we derive are uniform. Moreover, we will show that our bounds are optimal. Along the way, we also obtain novel results on the characterization of wavefront sets in 3 dimensions by compactly supported shearlets. Finally, geometric properties of ∂S such as curvature are described in terms of the continuous shearlet transform of f.

Keywords. Shearlets, Edge detection, Edge classification, Curvature, Higher dimensional shearlet transform, Wavefront set.

1 Introduction

One of the fundamental problems of imaging science is the extraction of edges from 2D and 3D images [2]. Edges in images are the most significant feature, which describes the shape of objects and also allows to infer information on the 3D spatial order [20]. In fact most of the information is contained along the edges of images. In medical imaging applications it is, for instance, an important task to separate or classify multiple, potentially overlapping structures. In order to do so, a careful analysis of the singularities and their geometric properties is necessary, see [21].

In this paper, we will address two aspects of analyzing singularities in images. First of all we describe the *detection* of singularities of an underlying 2D and 3D image, where – coarsely speaking – a singularity of an image describes a point in which the image is not smooth.

Second, if an edge is parametrized by a curve, it is, in particular from an information extraction point of view, important to describe the geometrical characteristics of this curve. Such characteristics include points p in which the curve does not possess a unique normal vector or the curvature in p. In particular, one asks for a *classification* of points on singularities.

Directional representation systems were in the past shown to be successful on certain aspects of this general task, in particular, curvelet and shearlet systems, see [1] and [18], respectively. In the sequel, we will focus our attention to shearlet systems, which are to date more extensively used, in particular, due to the fact that they provide a unified treatment of the continuous and digital realm, thereby enabling faithful numerical realizations [19].

1.1 Related Work

Historically first, it has been shown in [17] for a special shearlet system (and later extended in [7]), that shearlets are able to detect the wavefront set of a distribution in 2D. In terms of images, this implies that

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