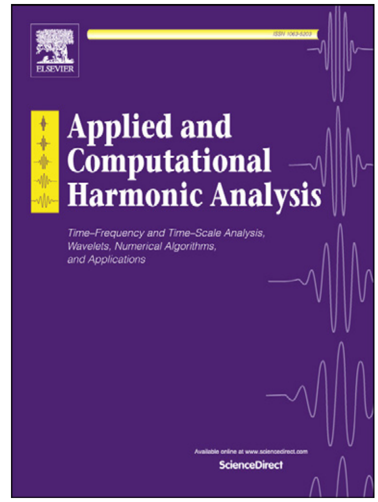

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Improved Recovery Guarantees for Phase Retrieval from Coded Diffraction Patterns

D. Gross[1,2], F. Krahmer[3], R. Kueng[*2,4]

[1]Institute for Theoretical Physics, University of Cologne
[2]Institute for Physics & FDM, University of Freiburg
[3] Department of Mathematics, Technische Universität München
[4]ARC Centre for Engineered Quantum Systems, School of Physics, The University of Sydney

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ABSTRACT. In this work we analyze the problem of phase retrieval from Fourier measurements with random diffraction patterns. To this end, we consider the recently introduced PhaseLift algorithm, which expresses the problem in the language of convex optimization. We provide recovery guarantees which require $\mathcal{O}(\log^2 d)$ different diffraction patterns, thus improving on recent results by Candès et al. [1], which require $\mathcal{O}(\log^4 d)$ different patterns.

1. INTRODUCTION

1.1. **The problem of phase retrieval.** In this work we are interested in the problem of *phase retrieval* which is of considerable importance in many different areas of science, where capturing phase information is hard or even infeasible. Problems of this kind occur, for example, in X-ray crystallography, diffraction imaging, and astronomy.

More formally, *phase retrieval* is the problem of recovering an unknown complex vector $x \in \mathbb{C}^d$ from *amplitude* measurements

$$y_i = |\langle a_i, x \rangle|^2 \quad i = 1, \ldots, m, \tag{1}$$

for a given set of measurement vectors $a_1, \ldots, a_m \in \mathbb{C}^d$. The observations y are insensitive to a global phase change $x \mapsto e^{i\phi}x$ – hence in the following, notions like "recovery" or "injectivity" are always implied to mean "up to a global phase". Clearly, the most fundamental question is: Which families of measurement vectors $\{a_i\}$ allow for a recovery of x in principle? I.e., for which measurements is the map $x \mapsto y$ defined by (1) injective?

Approaches based on algebraic geometry (for example [2, 3]) have established that for determining x, $4d + o(1)$ *generic* measurements are sufficient and $4d - \mathcal{O}(\log d)$ such observations are necessary. Here, "generic" means that the measurement ensembles for which the property fails to hold lie on a low-dimensional subvariety of the algebraic variety of all tight measurement frames.

This notion of generic success, however, is mainly of theoretical interest. Namely, injectivity alone neither gives an indication on how to recover the unique solution, nor is there any chance to directly generalize the results to the case of noisy measurements. It should be noted, however, that recently the notion of injectivity has been refined to capture aspects of stability with respect to noise [4].

Paralleling these advances, there have been various attempts to find tractable recovery algorithms that yield recovery guarantees. Many of these approaches are based on a linear reformulation in matrix space, which is well-known in convex programming. The crucial

*Corresponding author: richard.kueng@physik.uni-freiburg.de