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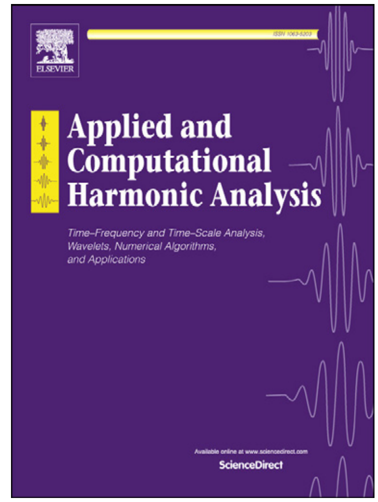

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Recovery Analysis for Weighted ℓ_1-Minimization Using the Null Space Property

Hassan Mansour, Rayan Saab

Abstract

We study the recovery of sparse signals from underdetermined linear measurements when a potentially erroneous support estimate is available. Our results are twofold. First, we derive necessary and sufficient conditions for signal recovery from compressively sampled measurements using weighted ℓ_1-norm minimization. These conditions, which depend on the choice of weights as well as the size and accuracy of the support estimate, are on the null space of the measurement matrix. They can guarantee recovery even when standard ℓ_1 minimization fails. Second, we derive bounds on the number of Gaussian measurements for these conditions to be satisfied, i.e., for weighted ℓ_1 minimization to successfully recover all sparse signals whose support has been estimated sufficiently accurately. Our bounds show that weighted ℓ_1 minimization requires significantly fewer measurements than standard ℓ_1 minimization when the support estimate is relatively accurate.

I. Introduction

The application of ℓ_1 norm minimization for the recovery of sparse signals from incomplete measurements has become standard practice since the advent of compressed sensing [1]–[3]. Consider an arbitrary k-sparse signal $x \in \mathbb{R}^N$ and its corresponding linear measurements $y \in \mathbb{R}^m$ with $m < N$, where y results from the underdetermined system

$$y = Ax. \tag{1}$$

It is possible to exactly recover all such sparse x from y by solving the ℓ_1 minimization problem

$$\min_z \ \|z\|_1 \text{ subject to } y = Az \tag{2}$$

if A satisfies certain conditions [1]–[3]. In particular, these conditions are satisfied with high probability by many classes of random matrices, including those whose entries are i.i.d. Gaussian random variables, when* $m \gtrsim k \log(N/k)$.

One property of the measurement matrix A that characterizes sparse recovery from compressive measurements is the null space property (NSP) (see, e.g., [4]) defined below.

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*We write $x \gtrsim y$ when $x \geq Cy$ for some constant independent of x and y.