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Spectral invariants in Lagrangian Floer homology of open subset ☆

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ABSTRACT

We define and investigate spectral invariants for Floer homology $HF(H, U : M)$ of an open subset $U \subset M$ in T^*M, defined by Kasturirangan and Oh as a direct limit of Floer homologies of approximations. We define a module structure product on $HF(H, U : M)$ and prove the triangle inequality for invariants with respect to this product. We also prove the continuity of these invariants and compare them with spectral invariants for the periodic orbits case in T^*M.

1. Introduction

Spectral invariants in cotangent bundles Spectral invariants in Symplectic Topology in terms of generating functions for Lagrangian submanifolds of cotangent bundles were introduced by Viterbo in [1]. If $E \to M$ is a smooth vector bundle over a compact smooth manifold M, $S : E \to \mathbb{R}$ a generic smooth function and

$$\Sigma_S := \{e \in E \mid d_{vert}S(e) = 0\}$$

(here $d_{vert}S$ denotes the derivative along the fibre), then

$$i_S : \Sigma_S \to T^*M, \qquad i_S(e) := dS(e)$$

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is a smooth Lagrangian immersion. It is known that all Hamiltonian deformations of the zero section can be generated by some function S in this way [2–4]. Viterbo defined spectral invariants as certain minimax values of S. He used them to prove several important results about Hamiltonian diffeomorphisms.

In [5,6] Oh defined spectral invariants for the case of cotangent bundles using the "homologically visible" critical values of the action functional

$$a_H(x) := \int_x \theta - \int_0^1 H(x(t), t)dt.$$

Here $x \in C^\infty([0,1], T^*M)$ is a smooth path satisfying $x(0), x(1) \in O_M$, O_M is the zero section and θ is the Liouville 1-form on T^*M.

More precisely, let $L = \phi_H^1(O_M)$, where ϕ_H^1 is a time-one-map generated by a Hamiltonian H. Let $HF_*^\lambda(O_M, \phi_H^1(O_M))$ denote the filtrated homology defined via the filtrated Floer complex:

$$CF_*^\lambda(O_M, \phi_H^1(O_M)) := \mathbb{Z}_2\langle\{x \in \mathrm{Crit}(a_H) \mid a_H(x) < \lambda\}\rangle.$$

These homology groups are well defined since the boundary map preserves the filtration:

$$\partial : CF_*^\lambda(O_M, \phi_H^1(O_M)) \to CF_*^\lambda(O_M, \phi_H^1(O_M)),$$

due to the well defined action functional that decreases along its "negative gradient flows". For a singular homology class $\alpha \in H_*(M, \mathbb{Z}_2)$ define

$$\sigma(\alpha, H) := \inf\{\lambda \in \mathbb{R} \mid F_H(\alpha) \in \mathrm{Im}(\imath_*^\lambda)\}$$

where

$$\imath_*^\lambda : HF_*^\lambda(O_M, \phi_H^1(O_M)) \to HF_*(O_M, \phi_H^1(O_M))$$

is the homomorphism induced by inclusion and

$$F_H : H_*(M) \to HF_*(O_M, \phi_H^1(O_M))$$

is the standard isomorphism defined by Floer between singular homology (modelled by Morse homology) and Floer homology groups (see [5] and the references therein).

The construction for spectral invariants in the case of a conormal bundle boundary condition is done in [5], and in [6] for cohomology classes. It turned out that Oh's invariants and those of Viterbo are in fact the same, see [7,8].

Oh proved in [5] that these invariants are independent of both the choice of almost complex structure J (which is used in the definition of Floer homology) and, after a certain normalization, the choice of H, as far as $\phi_H^1(O_M) = L$. Using these invariants $\sigma(\alpha, L) := \sigma(\alpha, H)$, Oh derived the non-degeneracy of Hofer's metric for Lagrangian submanifolds, a result earlier proved by Chekanov [9] using different methods. Another application to Hofer geometry is given in [10,11] in the characterization of geodesics in Hofer's metric for Lagrangian submanifolds of the cotangent bundle via quasi-autonomous Hamiltonians.

Spectral invariants in cotangent bundles were also studied by Monzner, Vichery and Zapolsky in [12].