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Spectral invariants in Lagrangian Floer homology of open subset $\stackrel{\mbox{\tiny\sc pr}}{\sim}$

ABSTRACT

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1. Introduction

Spectral invariants in cotangent bundles Spectral invariants in Symplectic Topology in terms of generating functions for Lagrangian submanifolds of cotangent bundles were introduced by Viterbo in [1]. If $E \to M$ is a smooth vector bundle over a compact smooth manifold $M, S : E \to \mathbb{R}$ a generic smooth function and

$$\Sigma_S := \{ e \in E \mid d_{vert} S(e) = 0 \}$$

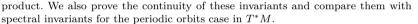
(here $d_{vert}S$ denotes the derivative along the fibre), then

 $i_S: \Sigma_S \to T^*M, \qquad i_S(e) := dS(e)$

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We define and investigate spectral invariants for Floer homology HF(H, U: M) of

an open subset $U \subset M$ in T^*M , defined by Kasturirangan and Oh as a direct limit

of Floer homologies of approximations. We define a module structure product on

HF(H, U: M) and prove the triangle inequality for invariants with respect to this

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is a smooth Lagrangian immersion. It is known that all Hamiltonian deformations of the zero section can be generated by some function S in this way [2–4]. Viterbo defined spectral invariants as certain minimax values of S. He used them to prove several important results about Hamiltonian diffeomorphisms.

In [5,6] Oh defined spectral invariants for the case of cotangent bundles using the "homologically visible" critical values of the action functional

$$a_H(x) := \int_x \theta - \int_0^1 H(x(t), t) dt.$$

Here $x \in C^{\infty}([0,1], T^*M)$ is a smooth path satisfying $x(0), x(1) \in O_M, O_M$ is the zero section and θ is the Liouville 1-form on T^*M .

More precisely, let $L = \phi_H^1(O_M)$, where ϕ_H^1 is a time-one-map generated by a Hamiltonian H. Let $HF_*^{\lambda}(O_M, \phi_H^1(O_M))$ denote the filtrated homology defined via the filtrated Floer complex:

$$CF_*^{\lambda}(O_M, \phi_H^1(O_M)) := \mathbb{Z}_2 \langle \{ x \in \operatorname{Crit}(a_H) \mid a_H(x) < \lambda \} \rangle.$$

These homology groups are well defined since the boundary map preserves the filtration:

$$\partial: CF_*^{\lambda}(O_M, \phi_H^1(O_M)) \to CF_*^{\lambda}(O_M, \phi_H^1(O_M)),$$

due to the well defined action functional that decreases along its "negative gradient flows". For a singular homology class $\alpha \in H_*(M, \mathbb{Z}_2)$ define

$$\sigma(\alpha, H) := \inf\{\lambda \in \mathbb{R} \mid F_H(\alpha) \in \operatorname{Im}(i_*^\lambda)\}$$

where

$$\imath^{\lambda}_{*}: HF^{\lambda}_{*}(O_{M}, \phi^{1}_{H}(O_{M})) \to HF_{*}(O_{M}, \phi^{1}_{H}(O_{M}))$$

is the homomorphism induced by inclusion and

$$F_H: H_*(M) \to HF_*(O_M, \phi^1_H(O_M))$$

is the standard isomorphism defined by Floer between singular homology (modelled by Morse homology) and Floer homology groups (see [5] and the references therein).

The construction for spectral invariants in the case of a conormal bundle boundary condition is done in [5], and in [6] for cohomology classes. It turned out that Oh's invariants and those of Viterbo are in fact the same, see [7,8].

Oh proved in [5] that these invariants are independent of both the choice of almost complex structure J(which is used in the definition of Floer homology) and, after a certain normalization, the choice of H, as far as $\phi_H^1(O_M) = L$. Using these invariants $\sigma(\alpha, L) := \sigma(\alpha, H)$, Oh derived the non-degeneracy of Hofer's metric for Lagrangian submanifolds, a result earlier proved by Chekanov [9] using different methods. Another application to Hofer geometry is given in [10,11] in the characterization of geodesics in Hofer's metric for Lagrangian submanifolds of the cotangent bundle via quasi-autonomous Hamiltonians.

Spectral invariants in cotangent bundles were also studied by Monzner, Vichery and Zapolsky in [12].

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