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Shape preserving properties of univariate Lototsky-Bernstein operators

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Abstract: The main goal of this paper is to study shape preserving properties of univariate Lototsky-Bernstein operators $L_n(f)$ based on Lototsky-Bernstein basis functions. The Lototsky-Bernstein basis functions $b_{n,k}(x)$ $(0 \le k \le n)$ of order n are constructed by replacing x in the i^{th} factor of the generating function for the classical Bernstein basis functions of degree n by a continuous nondecreasing function $p_i(x)$, where $p_i(0) = 0$ and $p_i(1) = 1$ for $1 \le i \le n$. These operators $L_n(f)$ are positive linear operators that preserve constant functions, and a non-constant function $\gamma_n^p(x)$. If all the $p_i(x)$ are strictly increasing and strictly convex, then $\gamma_n^p(x)$ is strictly increasing and strictly convex as well. Iterates $L_n^M(f)$ of $L_n(f)$ are also considered. It is shown that $L_n^M(f)$ converges to $f(0) + (f(1) - f(0))\gamma_n^p(x)$ as $M \to \infty$. Like classical Bernstein operators, these Lototsky-Bernstein operators enjoy many traditional shape p-reserving properties. For every $(1, \gamma_n^p(x))-$convex function $f \in C[0, 1]$, we have $L_n(f; x) \ge f(x)$; and by invoking the total positivity of the system $\{b_{n,k}(x)\}_{0\le k\le n}$, we show that if f is $(1, \gamma_n^p(x))-$convex, then $L_n(f; x)$ is also $(1, \gamma_n^p(x))-$convex. Finally we show that if all the $p_i(x)$ are monomial functions, then for every $(1, \gamma_{n+1}^p(x))-$convex function f, $L_n(f; x) \ge L_{n+1}(f; x)$ if and only if $p_1(x) = \cdots = p_n(x) = x$.

Keywords: Lototsky-Bernstein operators; Fixed point; Iterates; Shape preserving; Total positivity

MSC: 41A20; 41A36; 41A35; 41A50

1 Introduction

The primary goal of this paper is to study the shape preserving properties of the Lototsky-Bernstein operators. The Lototsky-Bernstein operators are generalizations of the classical polynomial Bernstein operators. The classical polynomial Bernstein operators are defined by

$$B_n(f; x) := \sum_{k=0}^{n} f\left(\frac{k}{n}\right) b_k^n(x),$$

where $f \in C[0, 1]$ and $\{b_k^n(x), 0 \le k \le n\}$ denotes the Bernstein basis for the space of polynomials of degree at most n:

$$b_k^n(x) = \binom{n}{k} x^k (1 - x)^{n-k}, 0 \le k \le n.$$

The Bernsetin operators B_n have been the object of intense research, and have been generalized in several directions, for example, the $q-$Bernstein operators [26] and the $h-$Bernstein operators ([31]). The polynomials $B_n(f)$ converge uniformly to f, although the convergence might be very slow ([10], p.166). Moreover the operators B_n reduce the variation and preserve the shape of f. Also the derivative of $B_n(f)$ of a function of class C^1 converges uniformly to f' (see [10]). For all

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