

Accepted Manuscript

The growth of polynomials outside of a compact set –the Bernstein-Walsh inequality revisited

Klaus Schiefermayr

PII: S0021-9045(17)30089-8
DOI: <http://dx.doi.org/10.1016/j.jat.2017.07.007>
Reference: YJATH 5163

To appear in: *Journal of Approximation Theory*

Received date: 31 March 2017
Revised date: 27 June 2017
Accepted date: 24 July 2017

Please cite this article as: K. Schiefermayr, The growth of polynomials outside of a compact set –the Bernstein-Walsh inequality revisited, *Journal of Approximation Theory* (2017), <http://dx.doi.org/10.1016/j.jat.2017.07.007>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



The Growth of Polynomials Outside of a Compact Set – the Bernstein-Walsh Inequality Revisited

Klaus Schiefermayr*

Abstract

In this paper, we present a new and simple proof of the classical Bernstein-Walsh inequality. Based on this proof, we give some improvements for this inequality in the case that the corresponding compact set is real.

Mathematics Subject Classification: 30C10, 30E10, 41A17, 41A50, 30C85

Keywords: Bernstein-Walsh inequality, Green function, Logarithmic capacity, Inverse polynomial image

1 The Bernstein-Walsh Inequality

Let K be a compact set in the complex plane \mathbb{C} with logarithmic capacity $\text{cap } K > 0$. Without loss of generality, we assume that K is such that $\overline{\mathbb{C}} \setminus K$ is connected, where $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ denotes the extended complex plane. Let $g_K(z)$ denote the Green function (with pole at ∞) for $\overline{\mathbb{C}} \setminus K$ and define $g_K(z) := 0$ for $z \in K$. Furthermore, let \mathbb{P}_n denote the set of all polynomials of degree n with complex coefficients and let $\|\cdot\|_K$ denote the supremum norm on K . Then the *Bernstein-Walsh inequality* (sometimes called *Bernstein-Walsh lemma* or *Bernstein lemma*), see [8, Theorem 5.5.7] or [11, Lemma 3.7] or [7, Section 12.1], reads as follows.

Theorem 1 (Bernstein-Walsh inequality). *For any polynomial $Q_n \in \mathbb{P}_n$,*

$$\frac{|Q_n(z)|}{\|Q_n\|_K} \leq e^{n \cdot g_K(z)} \quad (z \in \mathbb{C} \setminus K). \quad (1)$$

Note that inequality (1) is obviously also true for $z \in K$. Inequality (1) gives a very general upper bound for the modulus of a polynomial outside a compact set K with respect to its maximum value on K in terms of the corresponding Green function (which only depends on K). It is also common to consider inequality (1) with the notion of the level sets of the Green function $g_K(z)$. For each $R \geq 1$, let

$$\Gamma_R(K) := \{z \in \mathbb{C} : g_K(z) = \log R\} \quad (2)$$

define the *level set with index R* for the Green function $g_K(z)$ (note that $\Gamma_R(K) = K$ for $R = 1$). Then inequality (1) reads as

$$\frac{|Q_n(z)|}{\|Q_n\|_K} \leq R^n \quad (z \in \Gamma_R(K)). \quad (3)$$

*University of Applied Sciences Upper Austria, Campus Wels, School of Engineering, Stelzhamerstrasse 23, 4600 Wels, Austria, KLAUS.SCHIEFERMAYR@FH-WELS.AT

Download English Version:

<https://daneshyari.com/en/article/5773712>

Download Persian Version:

<https://daneshyari.com/article/5773712>

[Daneshyari.com](https://daneshyari.com)